CSC589 Introduction to Computer Vision
Lecture 5

A brief review of matrices and vector

Bei Xiao

Last lecture

- Image histogram equalization
- Border effect and padding
- Image gradients

Today’s lecture

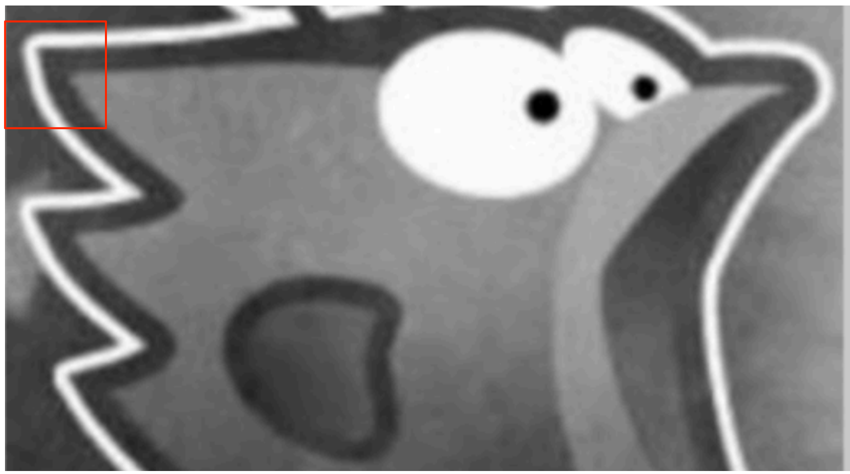
- A brief review of linear algebra
- Linear Algebra in Python

Take-home reading

- A review on matrix and vector of digital image processing (PDF will be attached in blackboard)
- 2D filtering with Python
- http://www.hdm-stuttgart.de/~maucher/Python/ComputerVision/html/Filtering.html

- Chapter 3.2

Grayscale Intensity Image as Matrices



row

The pixel P(i,j) has intensity value of 151

column

61	82	121	182	238	246	205	148	96	68	69
59	75	109	170	229	244	214	160	105	73	68
56	63	90	152	217	245	227	178	118	82	71
56	55	74	135	206	246	241	198	135	94	73
57	54	64	118	186	237	248	219	162	119	85
57	55	55	94	151	214	247	238	196	160	116
57	55	48	73	119	193	240	249	222	195	148
58	52	50	66	86	157	206	247	247	225	194
57	53	50	57	69	108	161	219	246	247	229
54	54	50	49	52	72	117	178	226	252	252
53	55	52	51	52	52	74	115	161	201	227

8 bit intensity image

- 8 bit graphics is a method of storing image information in a computer's memory or in an image file, such that each pixel is represented by one 8-bit type.
- The range of 8 bit intensity image is [0 255].

A mini tutorial on linear algebra

Basic Matrix Operations

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ldots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

ai,j represents the (i,j)

Basic Matrix Operations

Matrix products:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x1 \\ x2 \end{bmatrix} = \begin{bmatrix} ax_1 + bx_2 \\ cx_1 + dx_2 \end{bmatrix}$$

Vector dot product:

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \\ a_m \end{bmatrix}$$

Transpose of the column vector is a row vector

$$\mathbf{a}^T = [a_1, a_2, \cdots, a_m]$$

Dot product of two vectors (Algebraic definition)

$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \\ b_m \end{bmatrix}$$

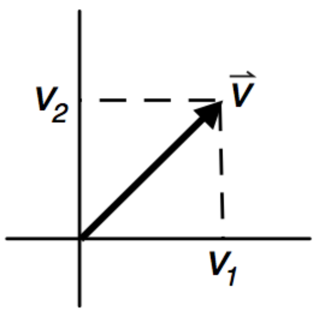
$$\mathbf{a}^T = [a_1, a_2, \cdots, a_m]$$

Dot product is a sum of pair-wise product of components

$$\begin{aligned} \mathbf{a}^T \mathbf{b} &= \mathbf{b}^T \mathbf{a} = a_1 b_1 + a_2 b_2 + \cdots + a_m b_m \\ &= \sum_{i=1}^{m} a_i b_i. \end{aligned}$$

What is a vector?

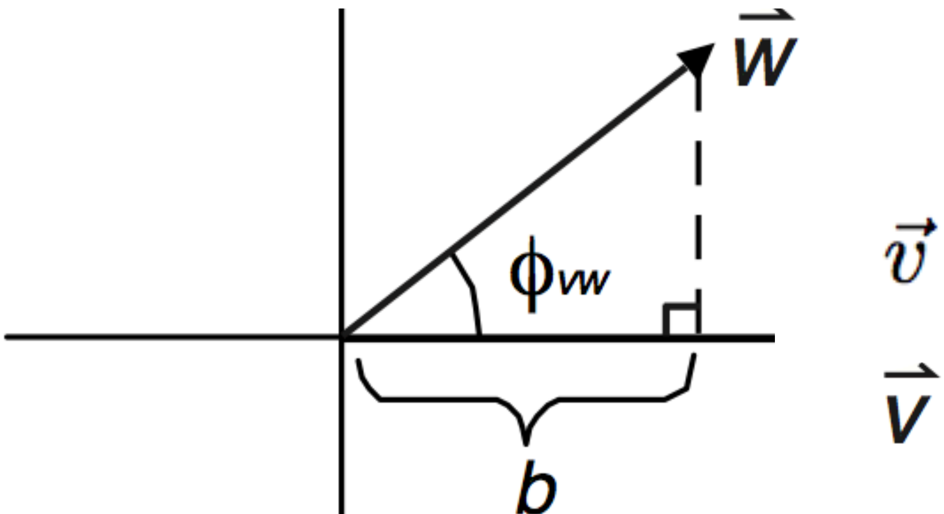
Vectors of dimension 2 or 3 can be graphically depicted as arrows, with the tail at the origin and the head at the coordinate location specific by the vector components.



Vector has a length and a direction. The norm or length is defined as:

$$\|v\| = \sqrt{\sum_{n} Vn^2}$$

Dot product (inner) of two vectors (Geometric definition)



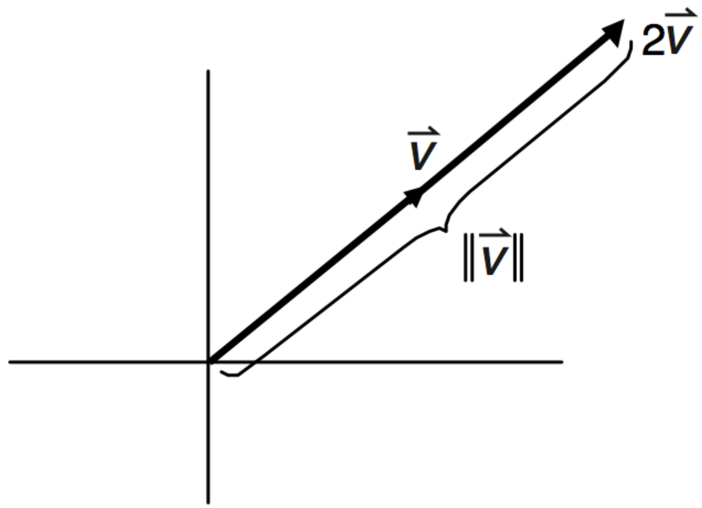
$$\vec{v} \cdot \vec{w} \equiv \|\vec{v}\| \, \|\vec{w}\| \, \cos\left(\phi_{vw}\right)$$

$$\vec{w} \cdot \vec{v} = \frac{b}{\|\vec{w}\|} \cdot \|\vec{w}\| \cdot \|\vec{v}\|$$

Φ is the angle between the two vectors

Scalar product

- Multiplying a vector by a scalar simply changes the length of the vector by that factor



Vector Space

- A vector space is a collection of vectors that is closed under linear combination.

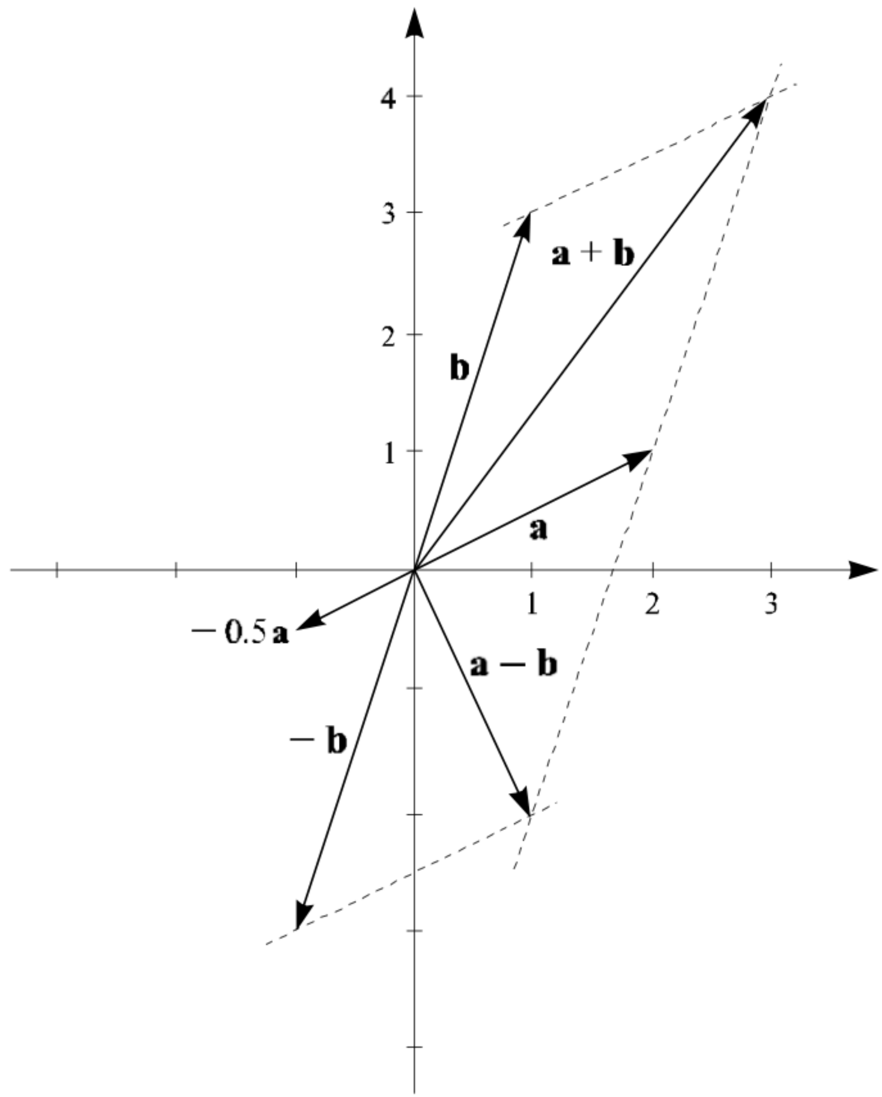
$$\mathbf{a} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\mathbf{a} + \mathbf{b} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

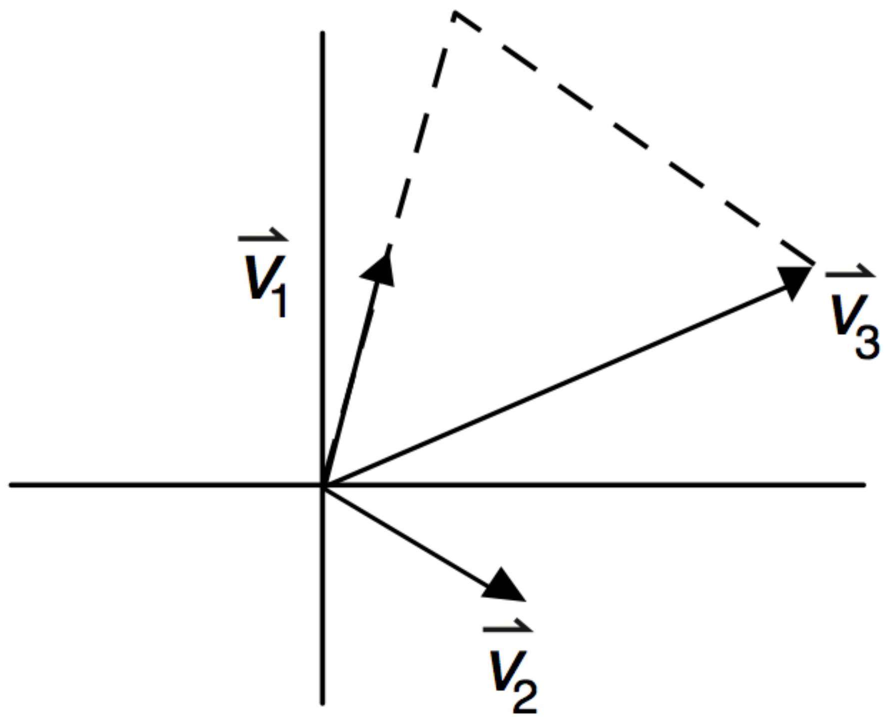
$$\mathbf{b} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

$$\mathbf{a} - \mathbf{b} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

Vector Space (geometric)



Vector Space (geometric)



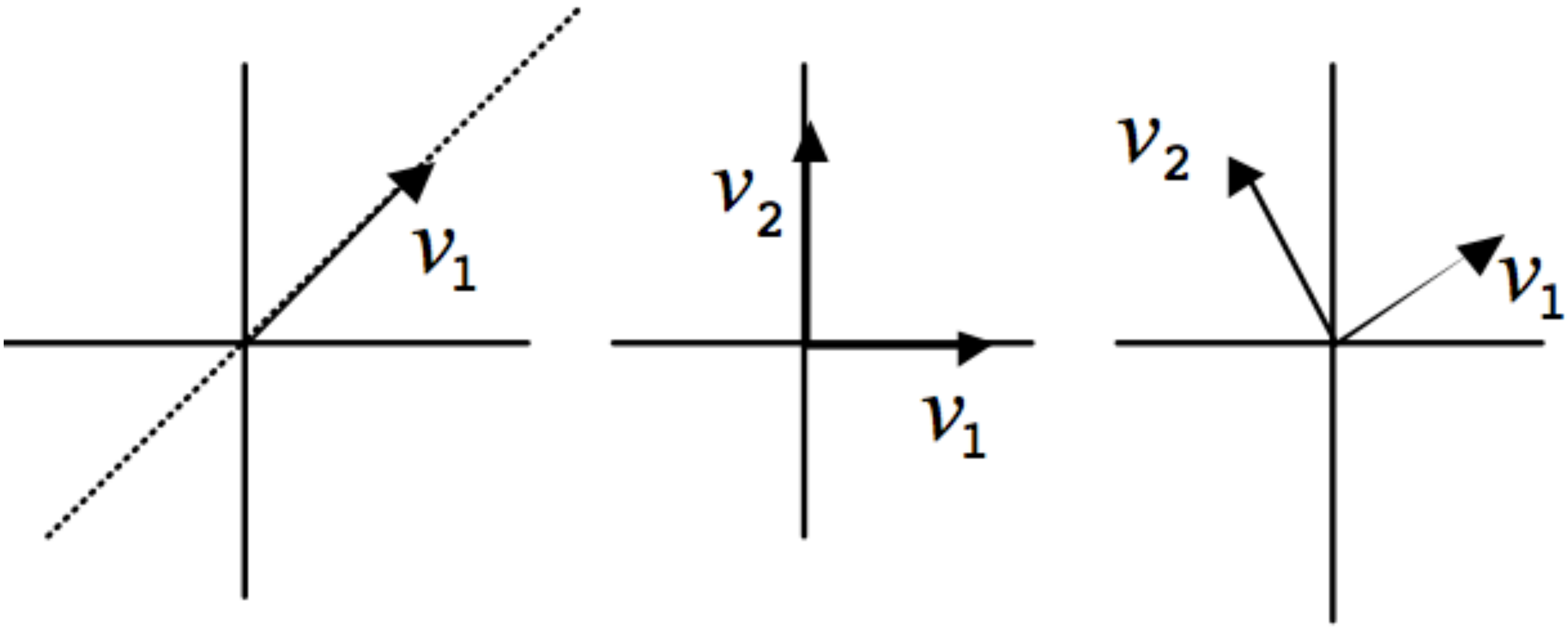
Basis

- A set of vectors in a vector space V is called a basis, or a set of basis vectors.
- A basis B of a vector space V is linearly independent if and only if:

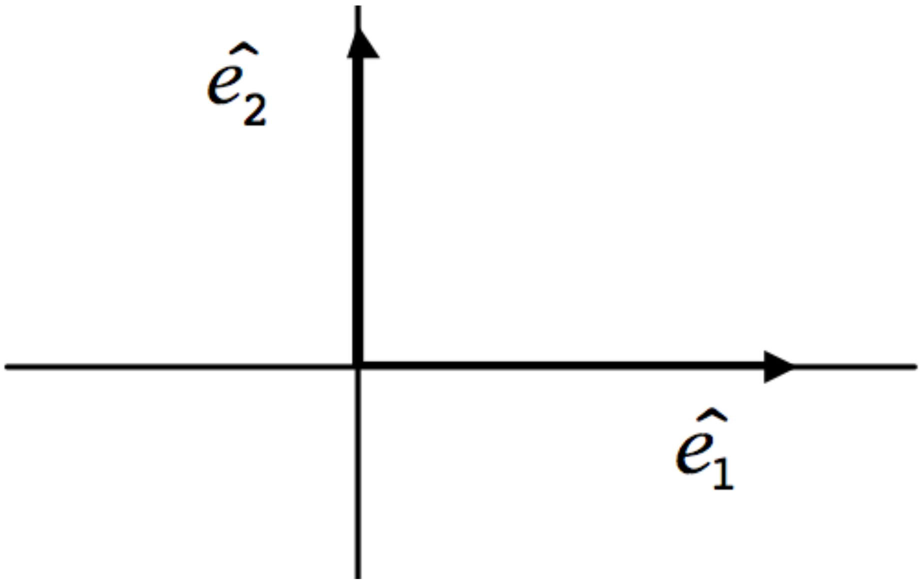
$$\sum_n \alpha_n \vec{v}_n = 0$$

- A basis for a vector space is linearly independent spanning set.

Basis vectors



Standard Basis (unit vectors)



$$\hat{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \; \hat{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \; \ldots \; \hat{e}_N = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}.$$

Linear Equations

$$
\begin{aligned}
a_{11}v_1 + a_{12}v_2 + \ldots + a_{1N}v_N &= \\
a_{21}v_1 + a_{22}v_2 + \ldots + a_{2N}v_N &= \\
&\vdots \\
{}_{M1}v_1 + a_{M2}v_2 + \ldots + a_{MN}v_N &=
\end{aligned}
$$

If we put the variables v_n and constant b_m into vectors and the constants am into a matrix A, these equations maybe written more compactly:

$$A\vec{v} = \vec{b}$$

To solve: $\vec{v} = A^{-1}b$

Matrix Inverse

- Matrix Inverse:

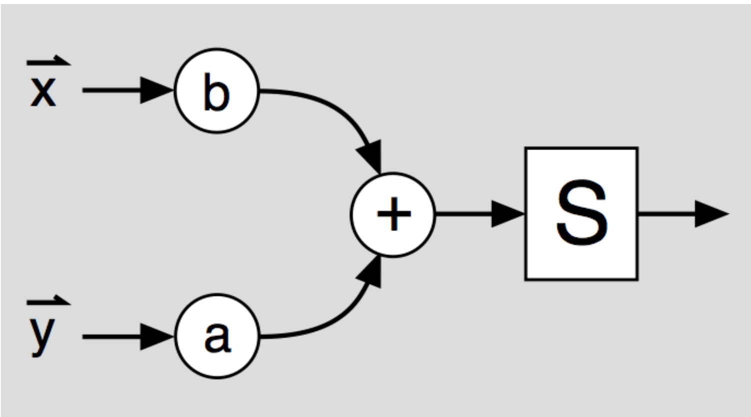
  $A^{-1}A = I$, I is identity matrix

$$I = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}.$$

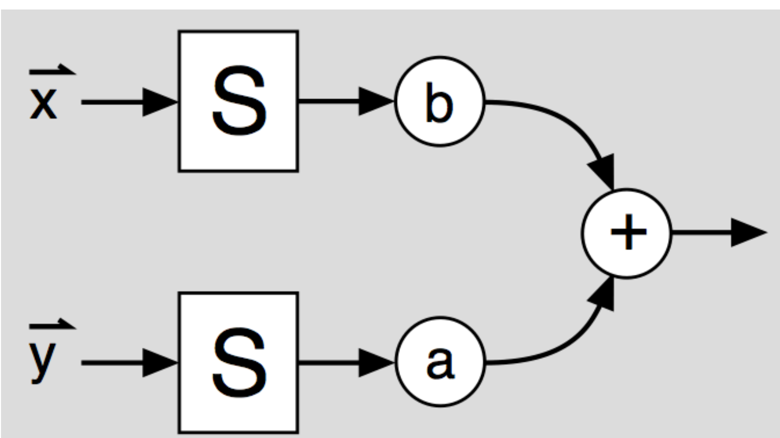
Inversion of a 2× 2 matrix

$$\mathbf{A}^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Linear System



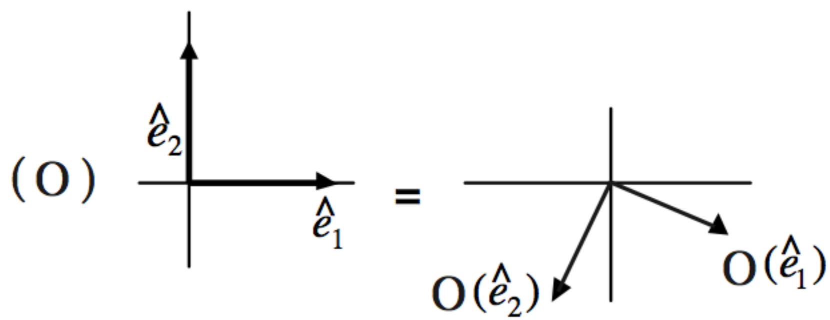
$$\mathcal{S}\{a\vec{v} + b\vec{w}\} = a\mathcal{S}\{\vec{v}\} + b\mathcal{S}\{\vec{w}\}.$$



A **linear system** S transforms vectors in one vector space into those of another vector space, in such a way that it obeys the principle of **superposition**:

Orthogonal marix

- $O^T O = I$, transpose of the matrix mulitplied by itself gives the identity matrix



$$e.g.\ O = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Linear algebra with Python

Arrays

```
import numpy as np
from numpy.linalg import *
# create a single array
 a = np.array([1, 4, 5, 8], float)
# create a multidimensional arary
 a = np.array([[1, 2, 3], [4, 5, 6]], float)
```

Quiz

- what is the result of the following matrix product:

$$\mathbf{A} = \begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 1 \end{bmatrix}.$$