CSC589 Introduction to Computer Vision
Lecture 6

Image Derivative, Image-Denoising

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Last lecture

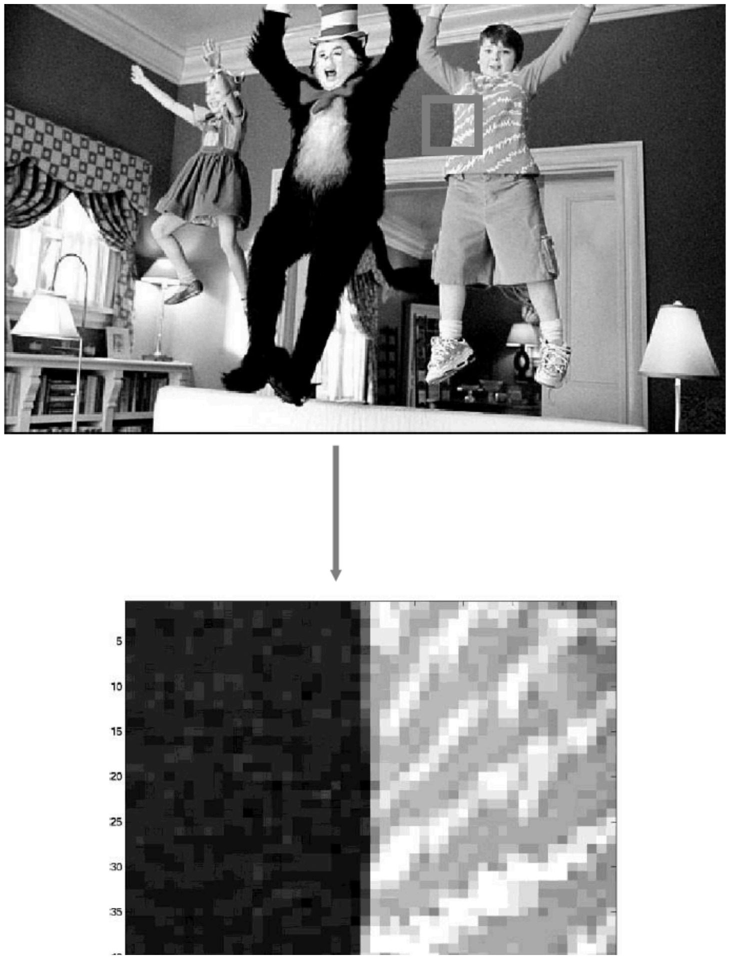
- Linear Algebra
- Matrix computation in Python

Today's lecture

- More on Image derivatives
- Quiz
- Image De-noising
- Median Filter
- Introduction to Frequency analysis

- Homework is due today! Please follow hand-in instructions. Be sure to include your write-up document!!

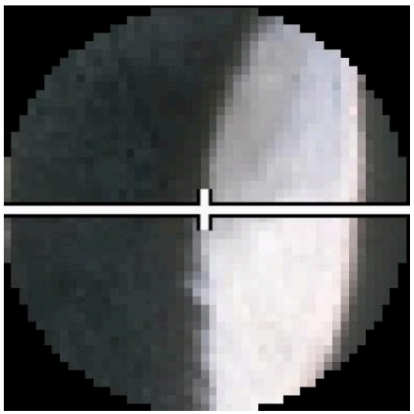
Compute Image Gradient

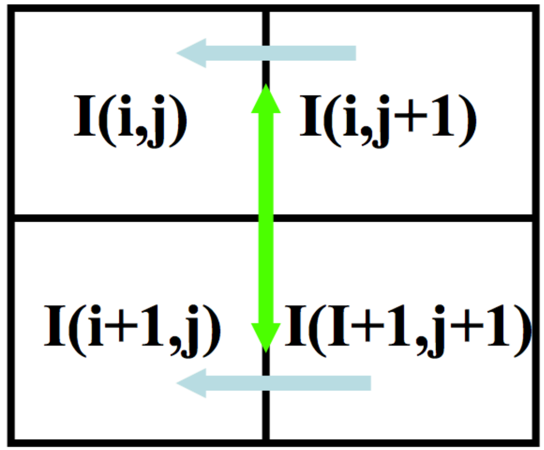


Example:

```
Is = imcrop(Ig);
Imagesc(Is);colormap(gray)
```

Compute gradient: first order derivatives





Compute gradient in the X-direction:

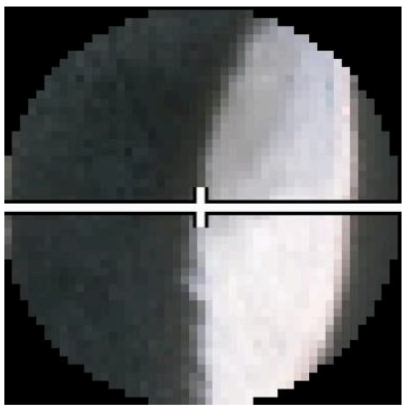
1) **Take the image intensity difference in the X-direction**
2) **Average the difference in the Y-direction(smoothing)**

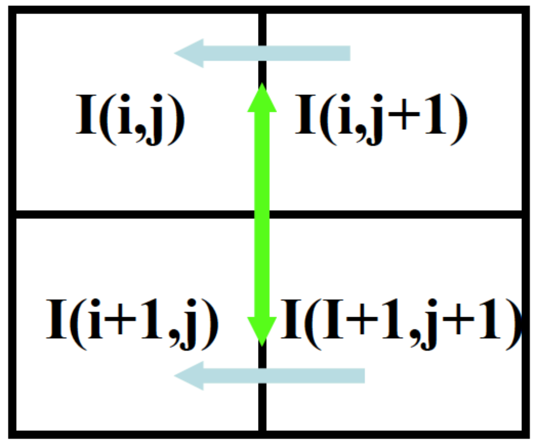
$$\frac{\delta I}{\delta x}(i,j) = \frac{1}{2}\left(\boxed{(I(i,j+1) - I(i,j))} + (\boxed{I(i+1,j+1) - I(i+1,j))}\right)$$

Slide source: Jianbo Shi

Compute gradient: first order derivatives

$$\frac{\delta I}{\delta x}(i,j) = \frac{1}{2}((I(i,j+1) - I(i,j)) + (I(i+1,j+1) - I(i+1,j)))$$

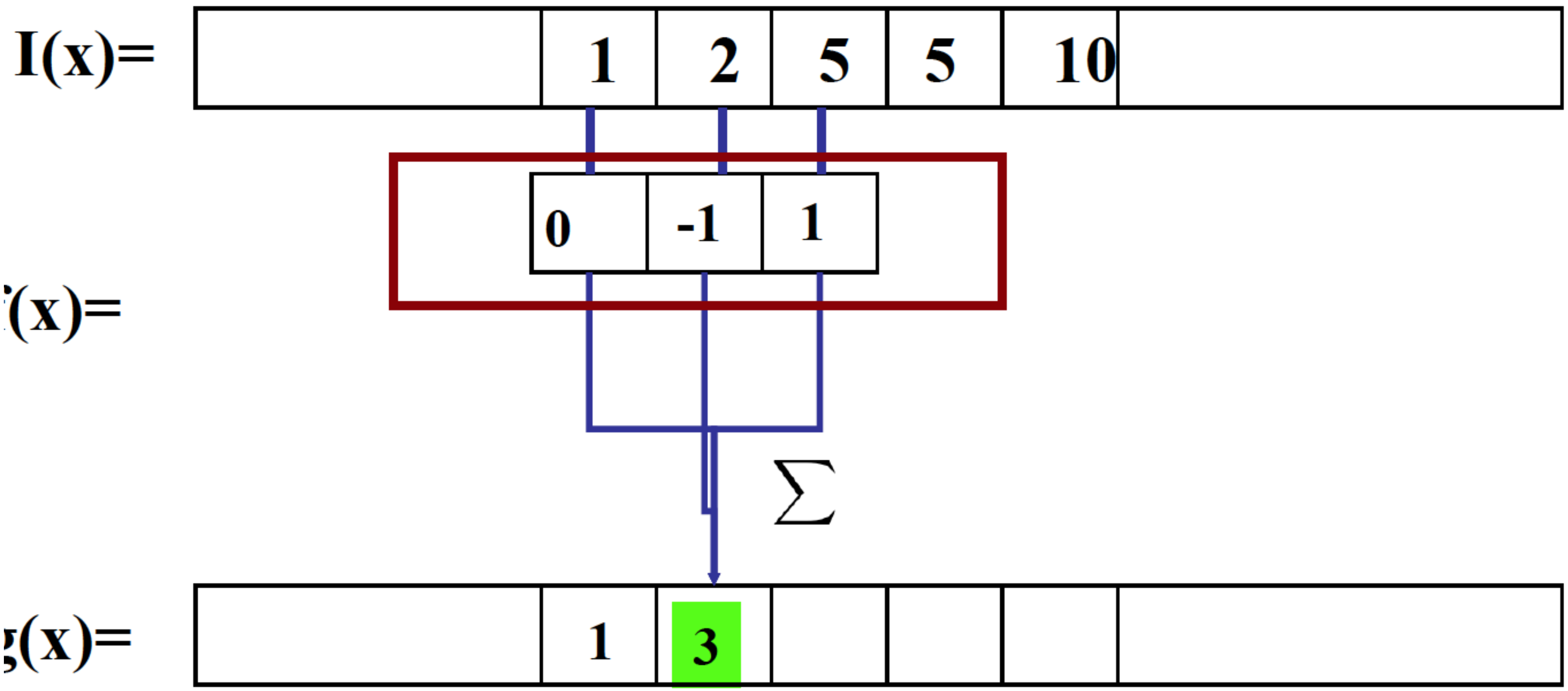




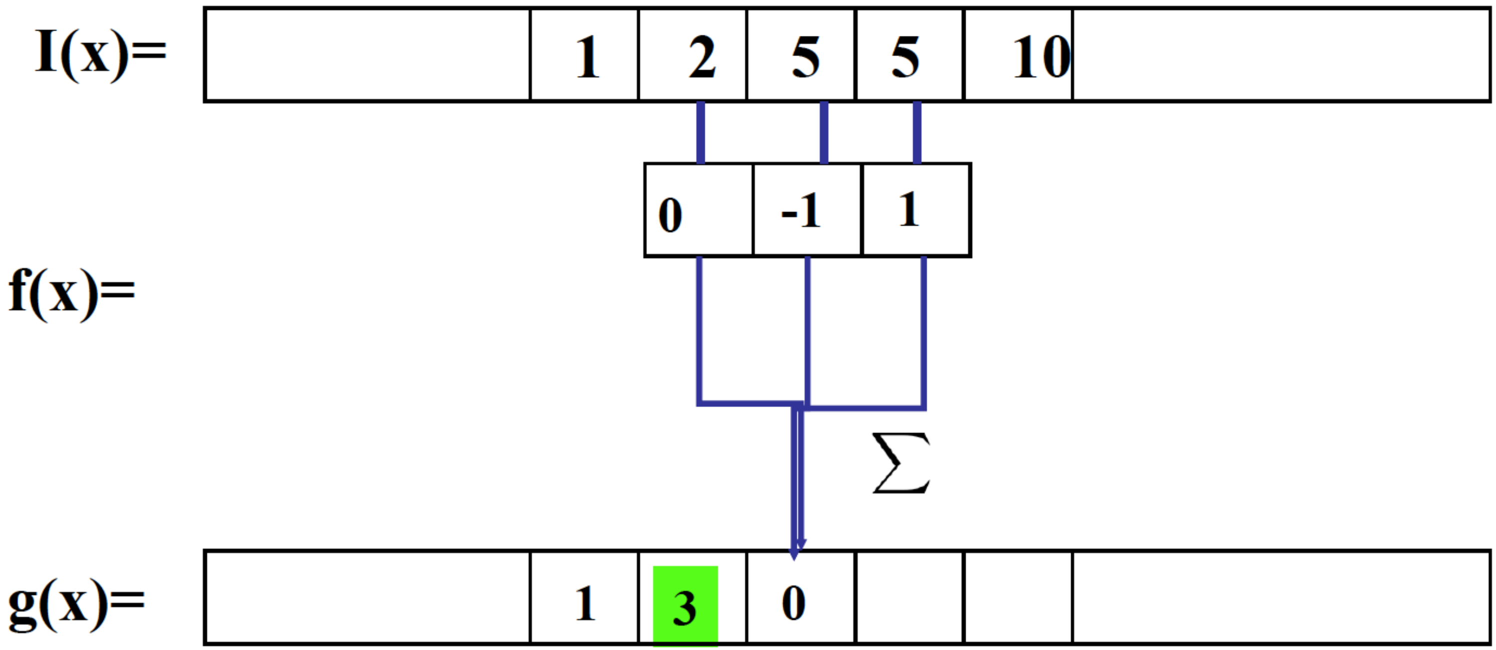

```
[nr,nc] = size(Is);
Ix = zeros(nr,nc);  % generate a empty matrix of size nr by nc
for i=1:nr-1,
    for j=1:nc-1,
        Ix(i,j) = 0.5*( (Is(i,j+1) - Is(i,j)) + (Is(i+1,j+1) - Is(i+1,j)));
    end
```

Slide source: Jianbo Shi

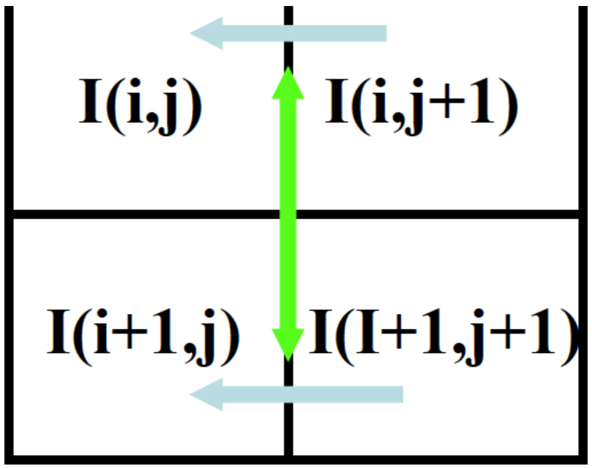
Compute gradient as convolution operation!



Compute gradient as convolution operation!



Compute gradient: first order derivatives

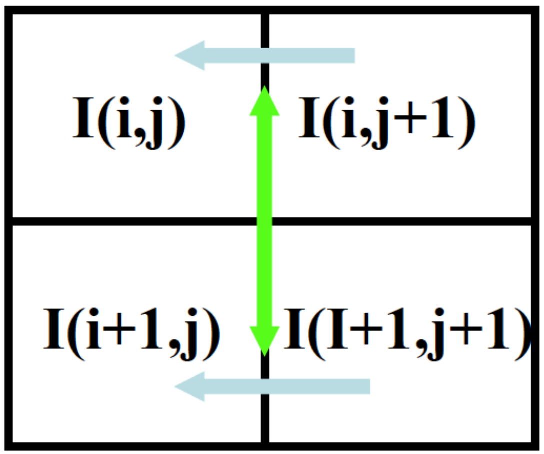


$$\frac{\delta}{\delta x} = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$\frac{\delta I}{\delta x}(i,j) = (I(i,j+1) - I(i,j));$$

$$= I \otimes (\frac{\delta}{\delta x})$$

Compute gradient: first order derivatives



$$\frac{\partial}{\delta x} = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$\mathbf{S} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

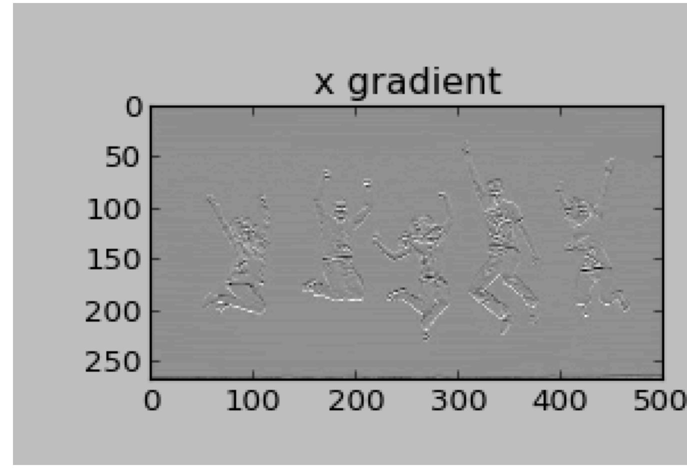
$$\frac{\delta I}{\delta x}(i,j) = \frac{1}{2}(((I(i,j+1) - I(i,j)) + (I(i+1,j+1) - I(i+1,j)))$$

$$= (I \otimes \frac{\delta}{\delta x}) \otimes S$$

Example

Usage in Python

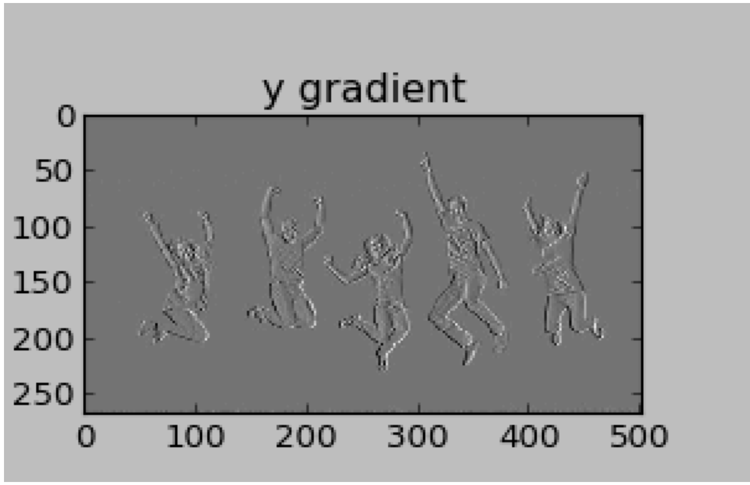
- s1 = np.array([1,1])
- dx = np.array([1,-1])
- dy = np.array([1,-1])
- x = ndimage.convolve1d(I,dx,axis= 0)
- gx_I = ndimage.convolve(x,s)



$$I_x = (I \otimes \frac{\delta}{\delta x}) \otimes S$$

Usage in Python

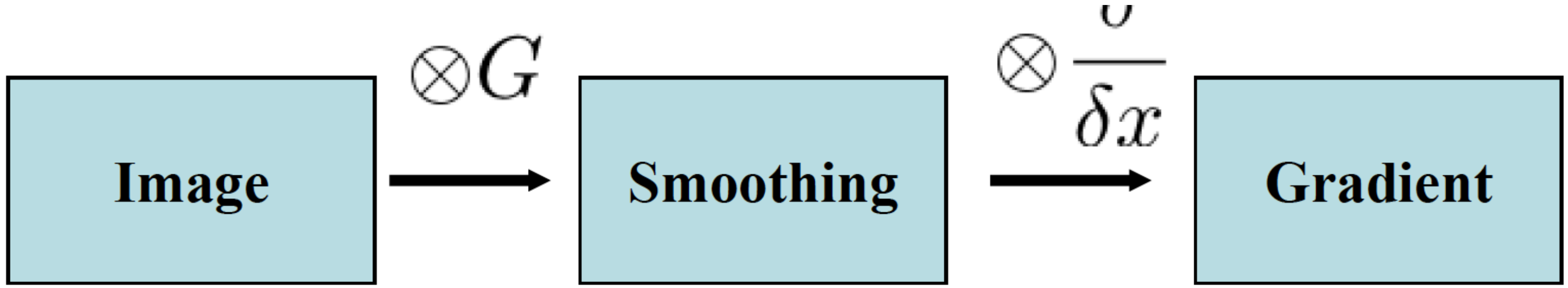
- s1 = np.array([1,1])
- dx = np.array([1,-1])
- dy = np.array([1,-1])
- y = ndimage.convolve1d(I,dx,axis= 1)
- gy_I = ndimage.convolve(y,s)



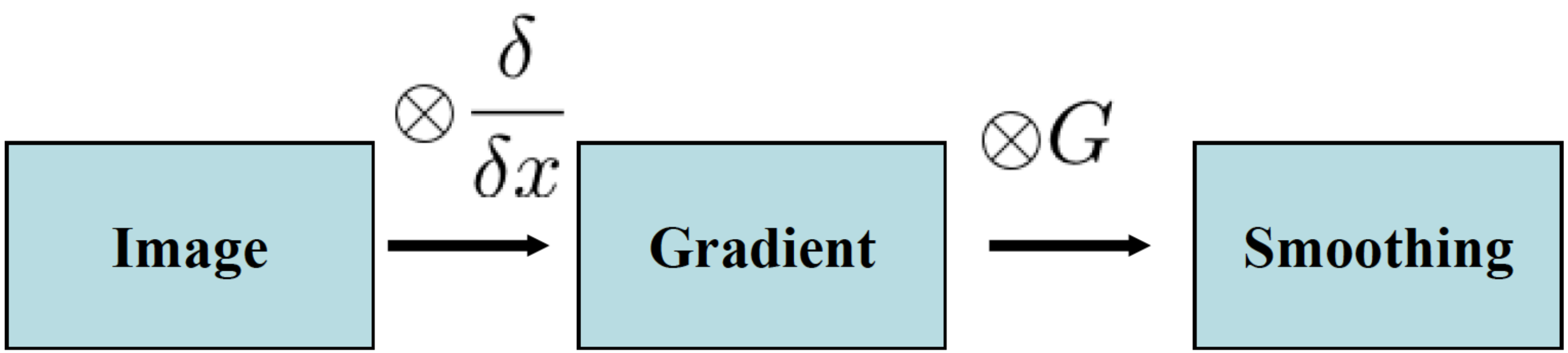
Or : gx_I,gy_I = np.gradient(I)[:2]

$$I_y = (I \otimes \frac{\delta}{\delta y}) \otimes S'$$

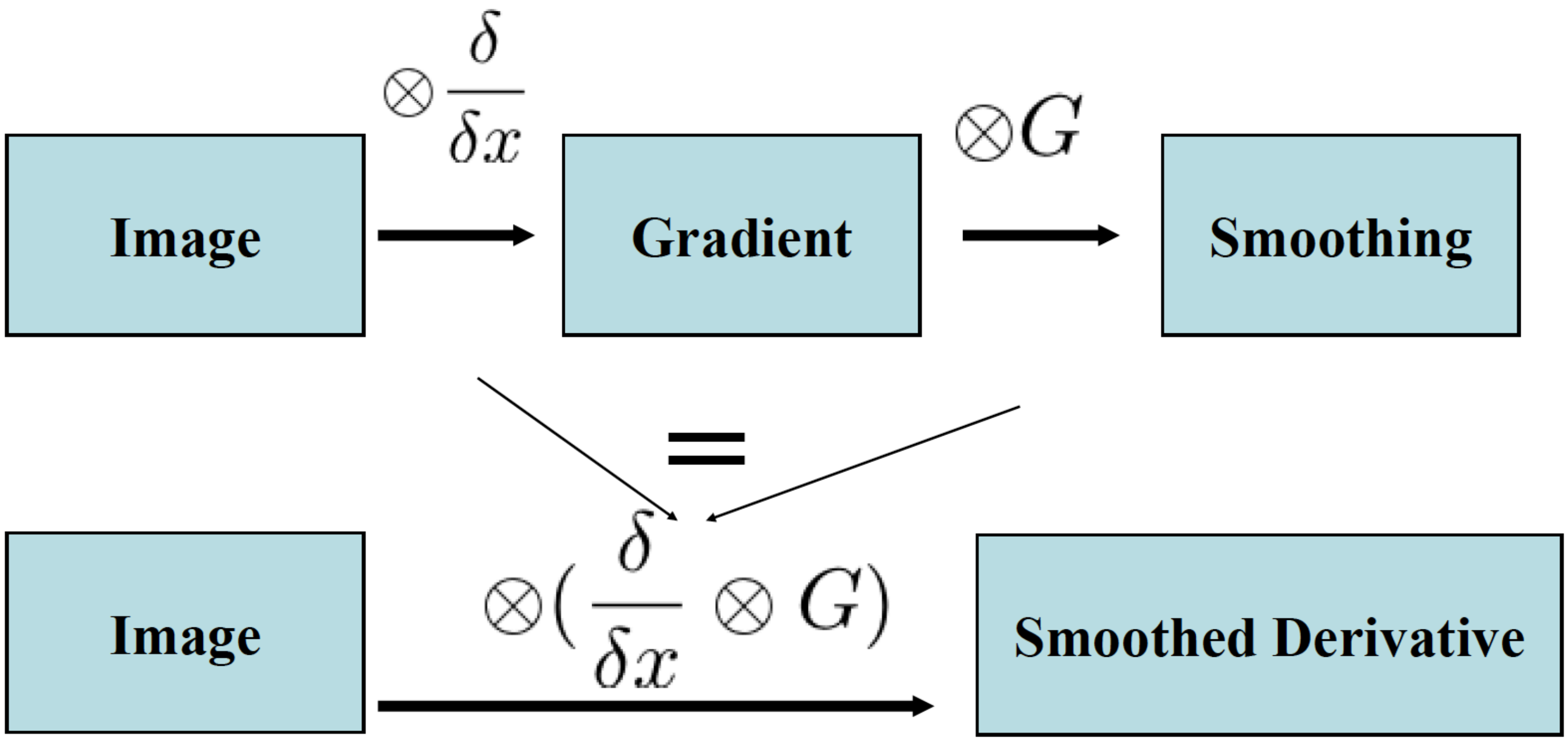
We can switch the order of smoothing and gradient



? =



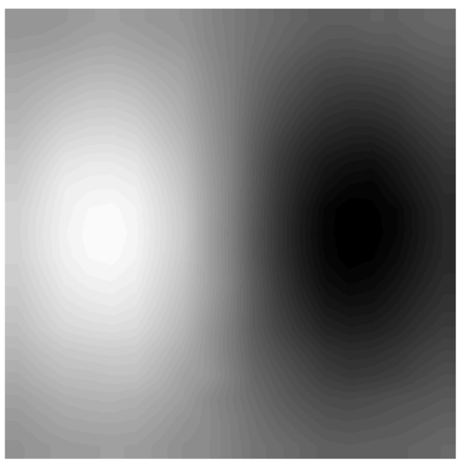
We can simplify even more



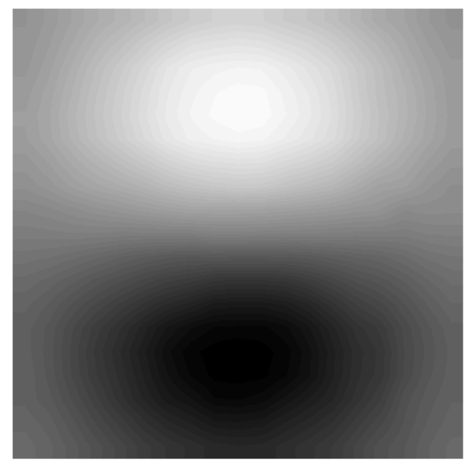
Smoothed derivative filter

$$\frac{\delta}{\delta x} \otimes G = \frac{\delta G}{\delta x} \longrightarrow \frac{\delta G}{\delta x} = -\frac{2x}{\sigma_x^2} G(x, y)$$

Gx



Gy



Sobel Filter

- Product of averaging and gradient.
- An cross product of two 1 d filter, Gaussian and gradient

$$\begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & +1 \end{bmatrix}$$

Review Questions (please turn in your answer)

1. Write down a 3×3 filter that returns a positive value if the average value of the 4-adjacent neighbors is less than the center and a negative value otherwise. Hint: don’t forget the normalization factor.
2. Write down a filter that will compute the gradient in the x-direction

   gradx(y,x) = im(y,x+1)-im(y,x) for each x,y

Review Questions (please turn in your answer)

Filtering Operator

3. Fill in the blanks:

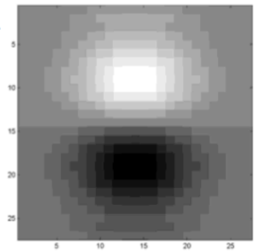
a) _ = D * B

b) A = _ * _

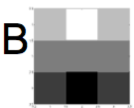
c) F = D * _

d) _ = D * D

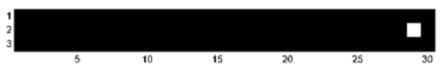
A



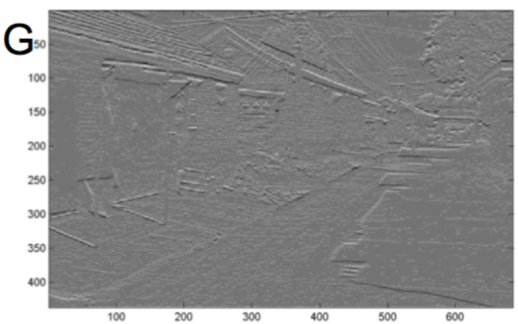
B



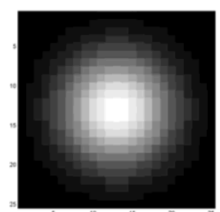
E



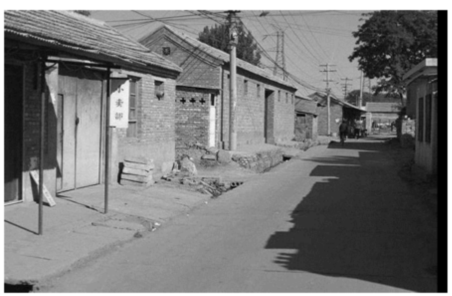
G



C

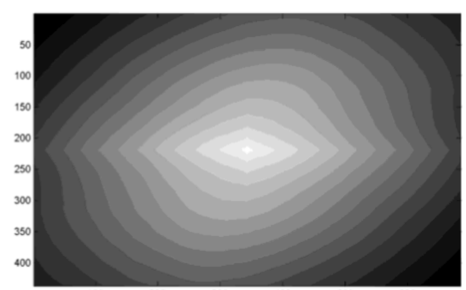


F



H

I



D

Slide: Hoiem

Image Noise

- Types of noises in images
  - Gaussian noise: Poor illumination, additive, independent for each pixel
  - Salt and Pepper: Dead pixels on LCD monitor
  - Film grain, poison distribution.

Image Noise

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Additive Gaussian noise

Salt and pepper noise

Image Noise

- Add Gaussian noise: Image + noise
- In Python:

```
noisy = I + 0.4 * I.std() *np.random.random(I.shape)
```

- Salt and Pepper noise
- Randomly replace pixels with white and black values
- In Python:

```
num_salt = np.ceil(0.05 * I.size * 0.5)
coords = [np.random.randint(0, i - 1, int(num_salt))
   for i in I.shape]
```

Median Filter

X = [2 80 6 3]

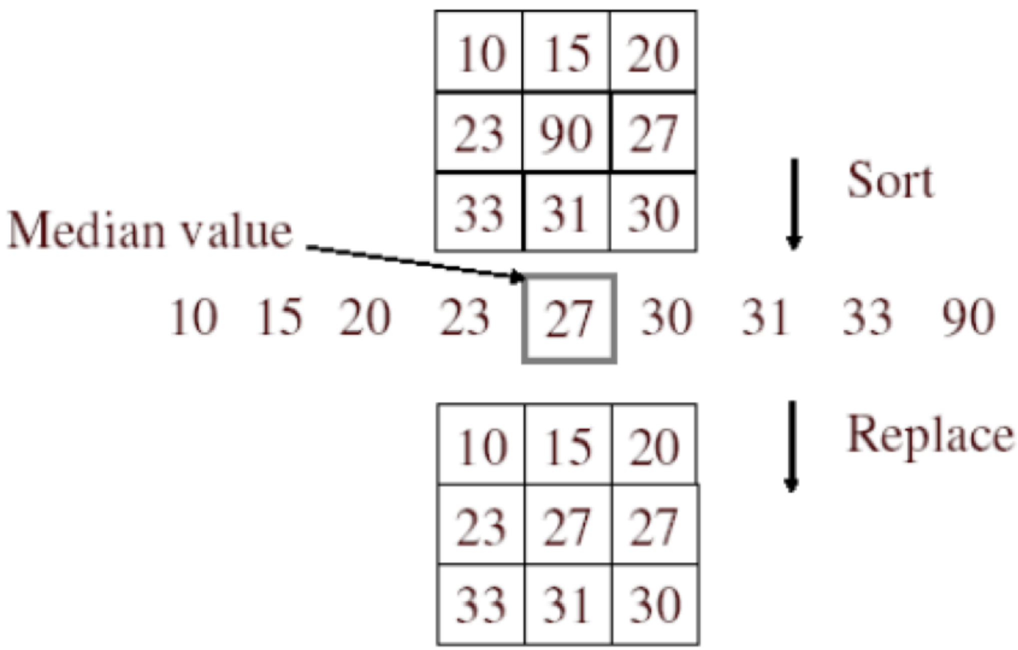
The median filter has a window size 3

The median filtered out signal y will be:
Y[1] = Median [2 2 80] = 2
Y[2] = Median [2 80 6] = 6
Y[3] = Median[80 6 3] = 6
Y[3] = Median [6 3 3] = 3

Notice the repeating of the first element

Selecting one pixel as a time; Not as efficient as Gaussian Filter

Median Filter

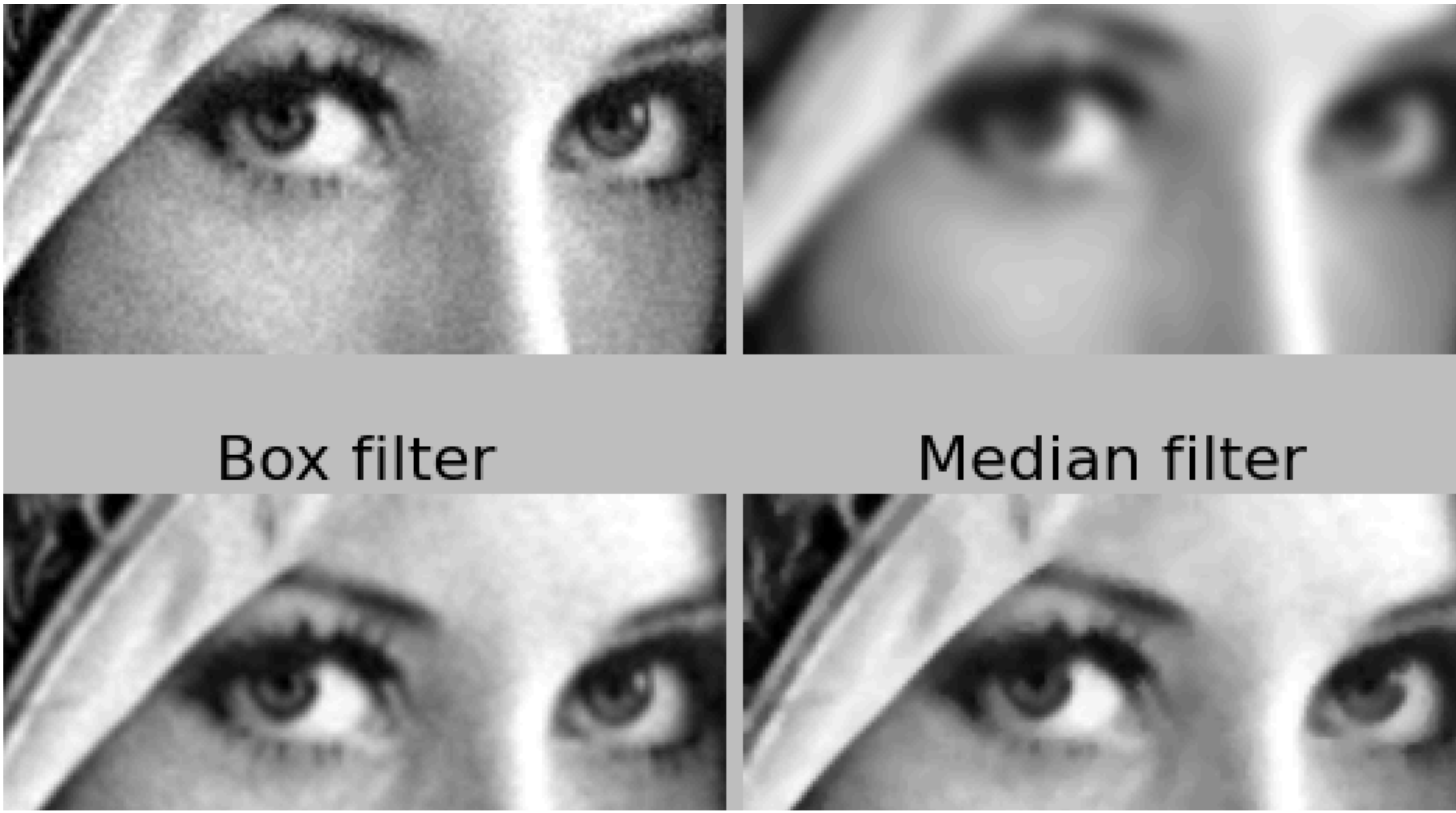


- No new pixel values introduced
- Removes spikes: good for impulse, salt & pepper noise
- Linear?

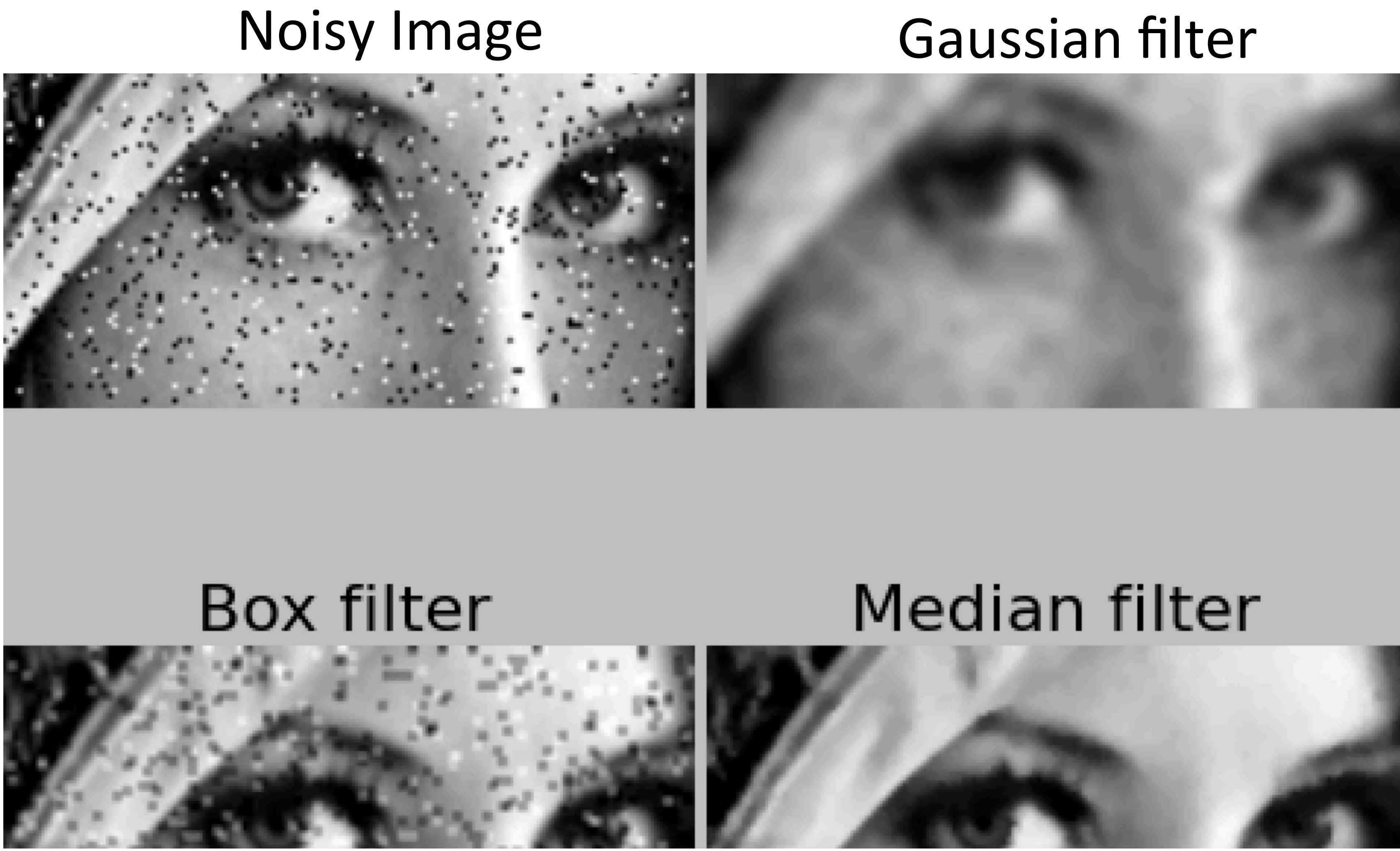
Comparison of the de-noisy results

Noisy Image

Gaussian filter



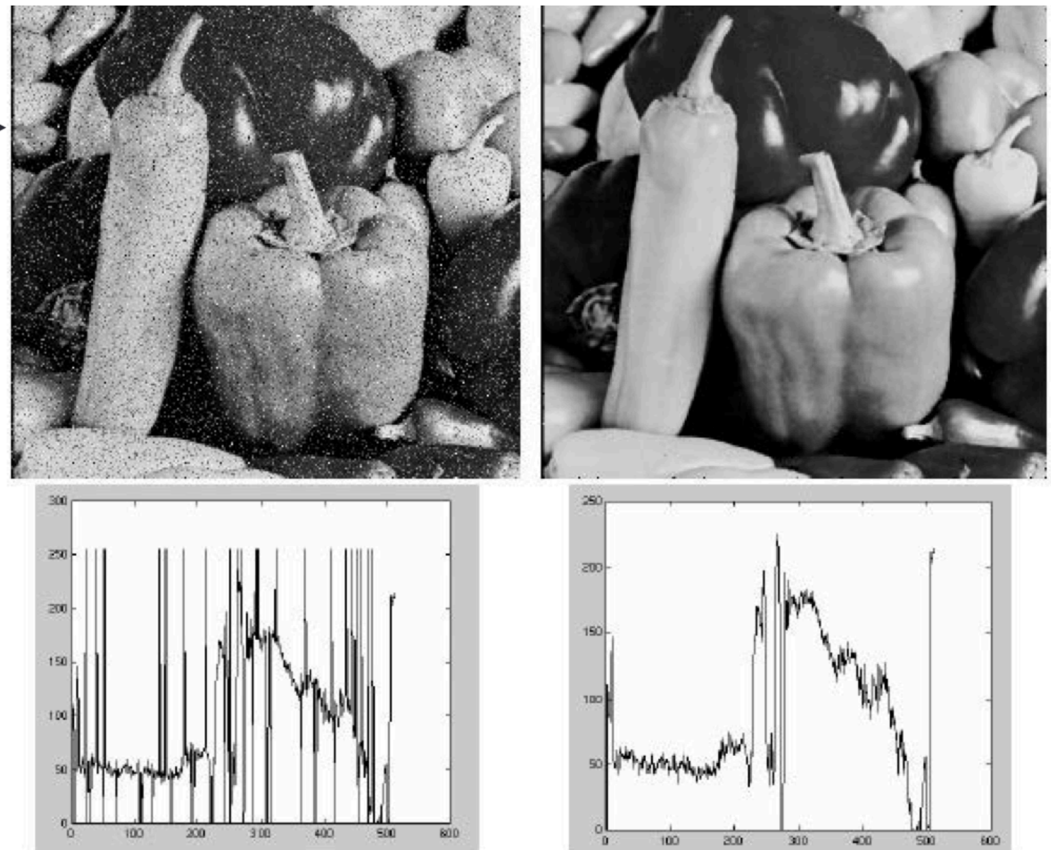
Comparison of the de-noisy results



Median Filter

Salt and pepper noise

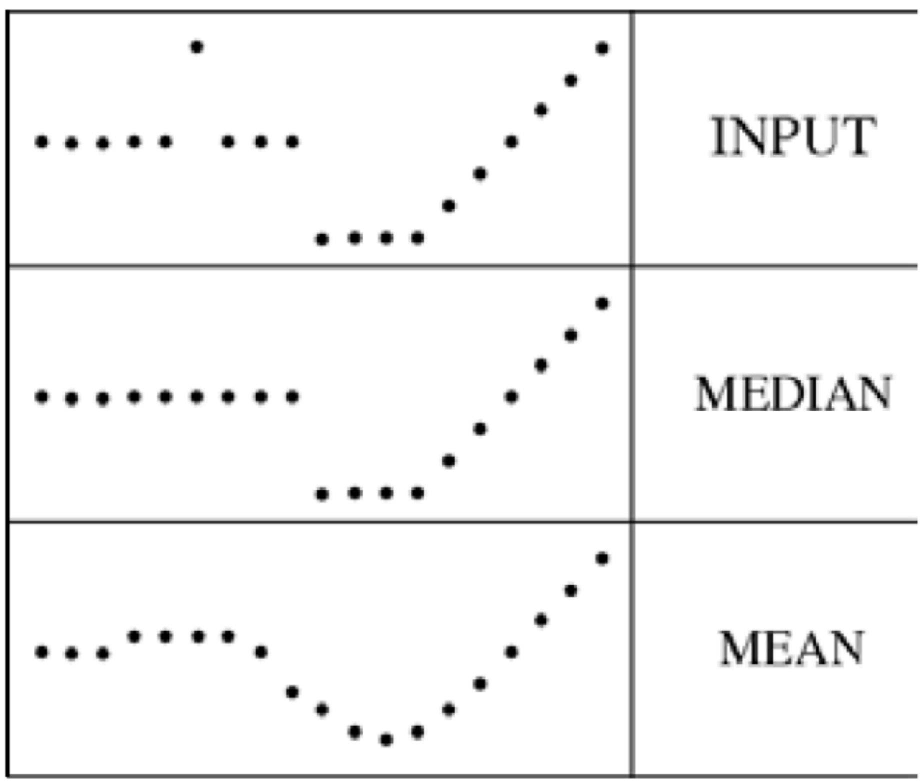
Median filtered



Plot of a row of the image

Median Filter

- Median filter is edge preserving



K. Grauman

Pros and Cons of median filter

- Pros:
  - The median is a more robust average than the mean and a single very unrepresentative pixel in a neighborhood will not affect the median value significantly.
  - The median value must actually be from the image pixels, so the median filter does not create new unrealistic pixel values when the filter straddles an edge.
- Cons:
  - selecting one pixel one time, not as efficient as Gaussian

Median Filter in Pyton

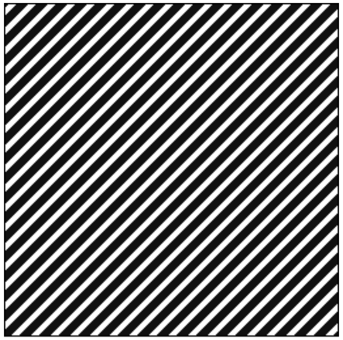
- med_denoised = ndimage.median_filter(noisy, windowsize)

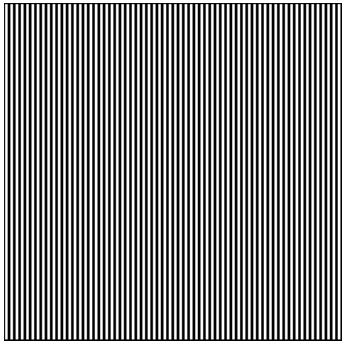
Exercise

- Use the following image (uploaded in blackboard) and explore the effect of median filtering with different neighborhood size

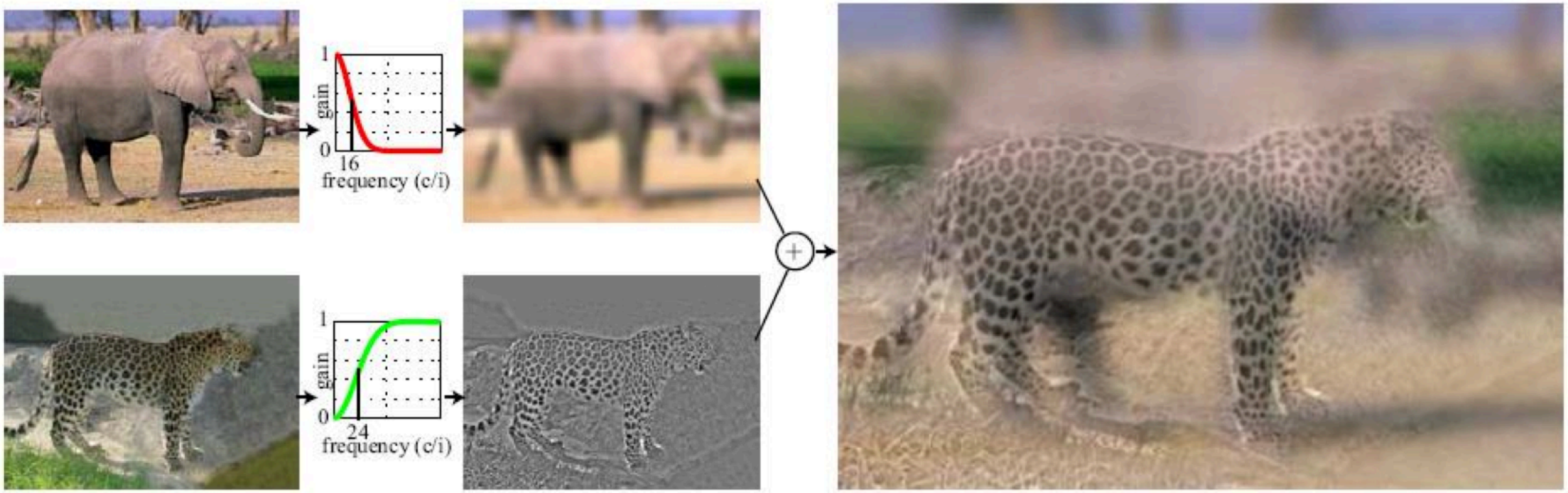
Exercise

- Unlike Gaussian filter, median filter is nonlinear.
- Median [A(x) + B(x)] = median[A(x)] + median[B(x)]
- Illustrate this to yourself by performing smoothing and pixel addition (in the order above) to a set of test images



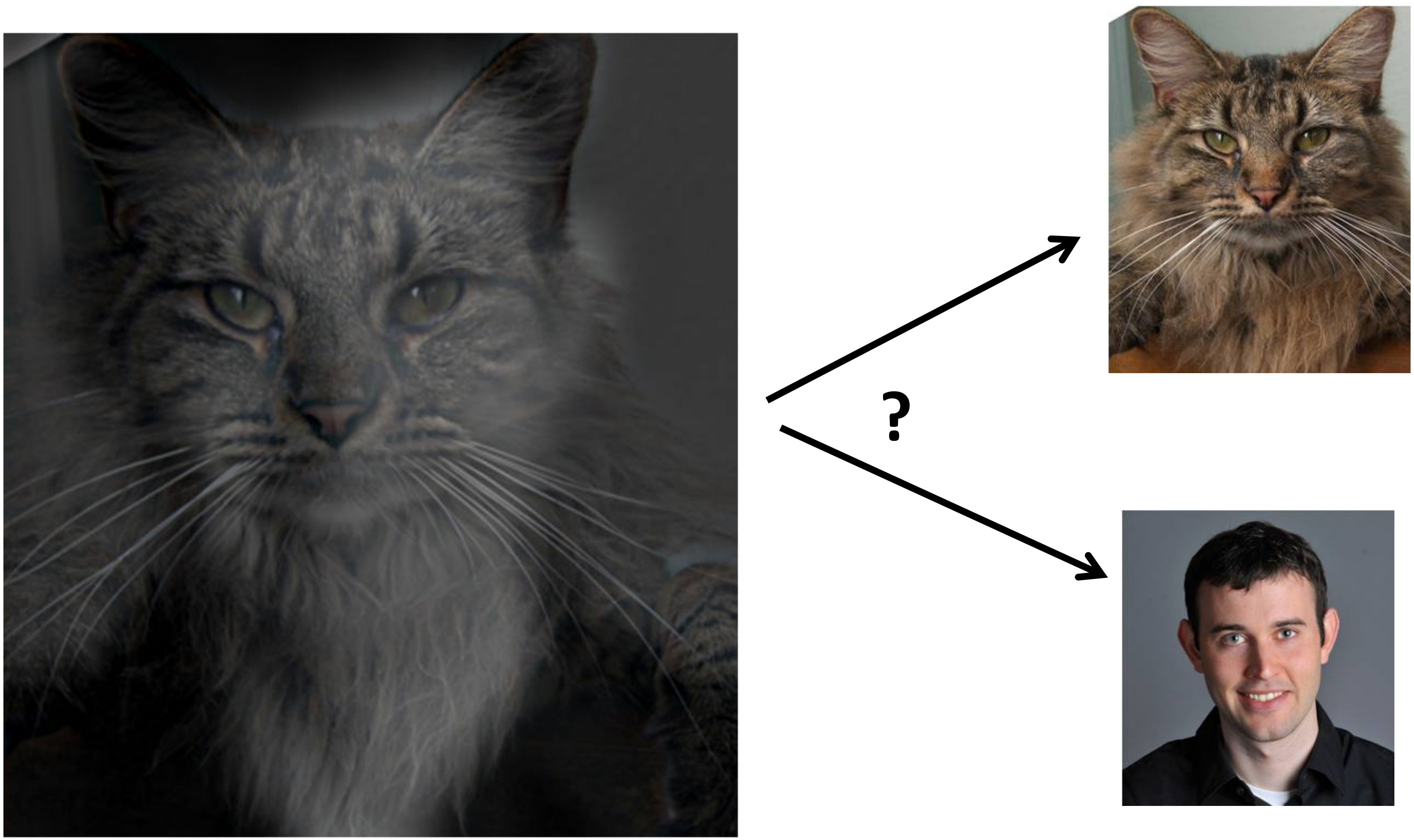


Hybrid Image



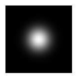
- A. Oliva, A. Torralba, P.G. Schyns, "Hybrid Images," SIGGRAPH 2006

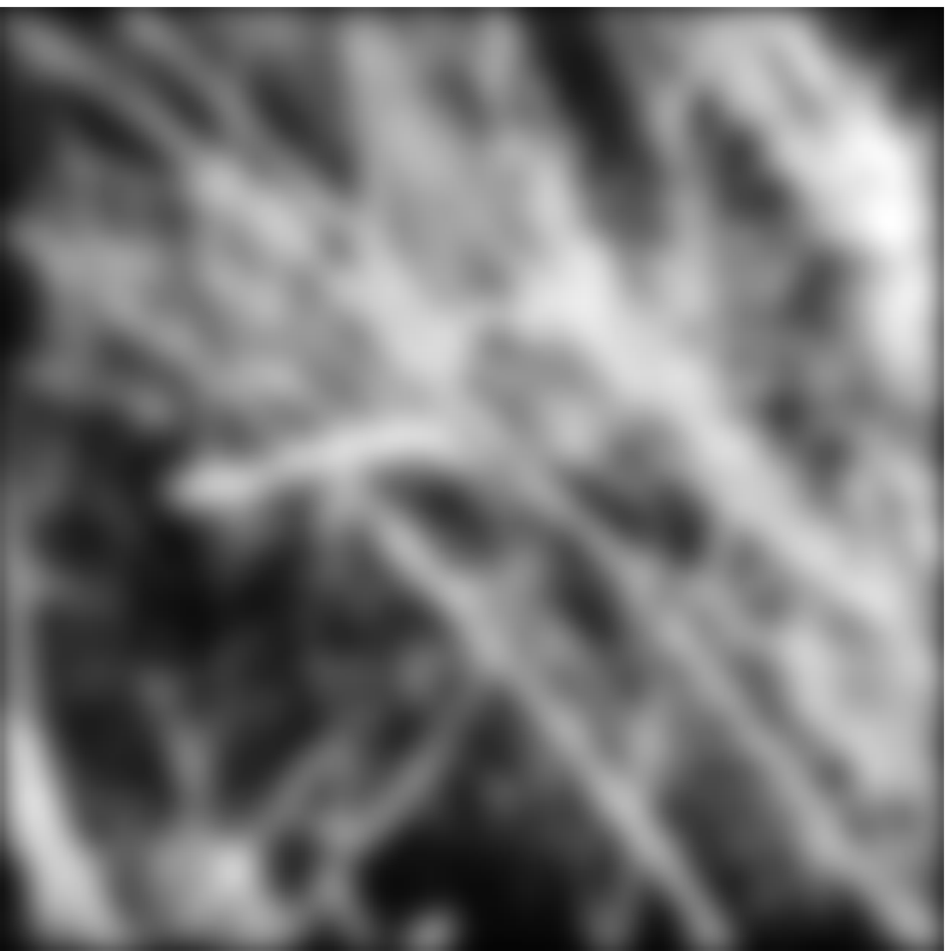
Why do we get different, distance-dependent interpretations of hybrid images?



Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

Gaussian





Box filter