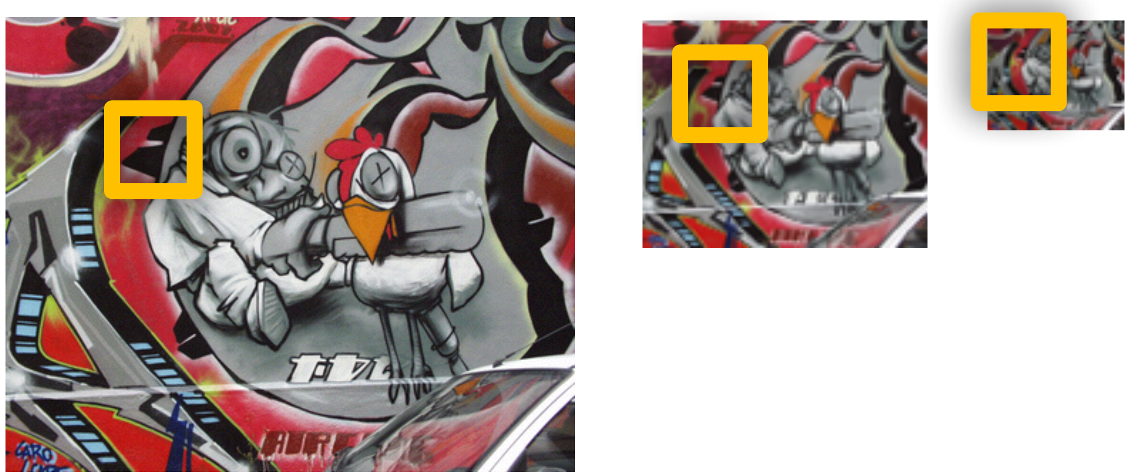
CSC 589 Introduction to Computer Vision

Lecture 16

More on Harris Corner Detection, Mid-term Review, and Blob Detection



Bei Xiao

Spring, 2014

American University

A bit of mid-term review

Questions: What is an unsharp mask? Is it a linear filter?

$$g_{\mathrm{unsharp}} = f(1 - \gamma h_{\mathrm{blur}} * f).$$

A bit of mid-term review

Questions: What is an unsharp mask? Is it a linear filter?

$$g_{\text{unsharp}} = f(1 - \gamma h_{\text{blur}} * f).$$

A bit of mid-term review

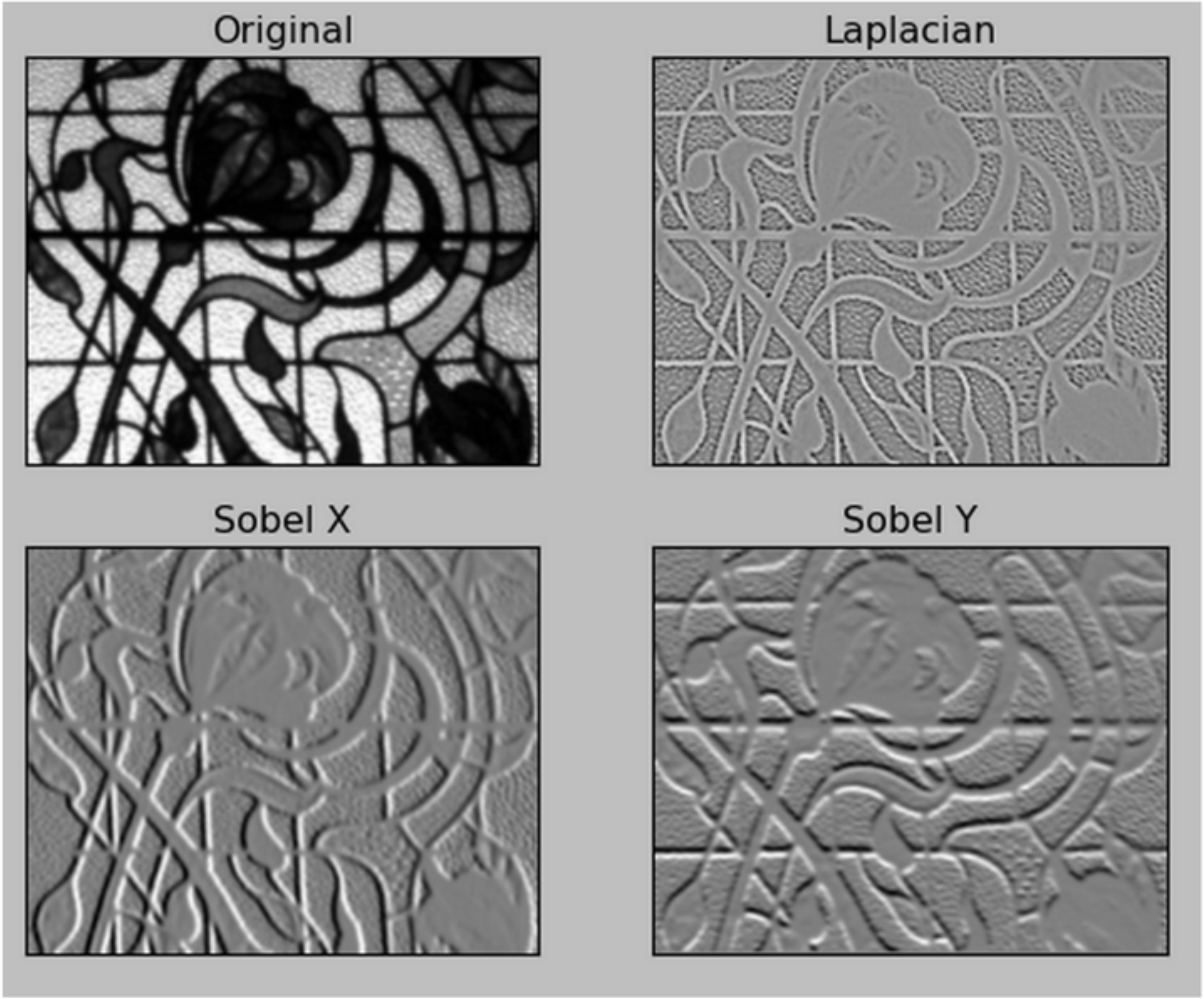
Questions: What is Difference of Gaussians? Laplacian kernel? What are their relationships? Can you plot the functions?

An example of laplacian kernel

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

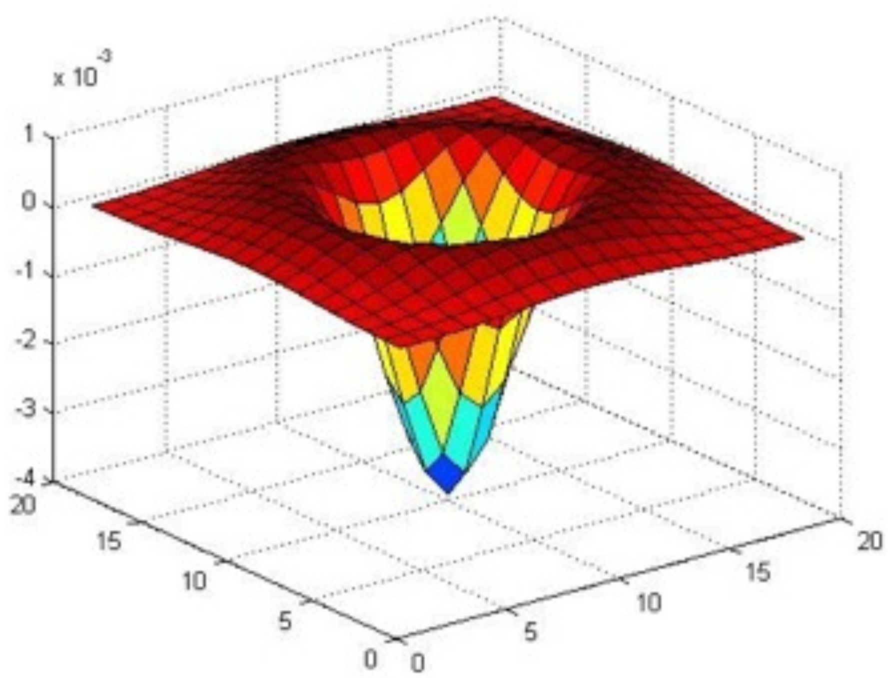
What happens if you apply the above kernel to images?
What is the difference between laplacian and sobel filter in terms of edge detection?

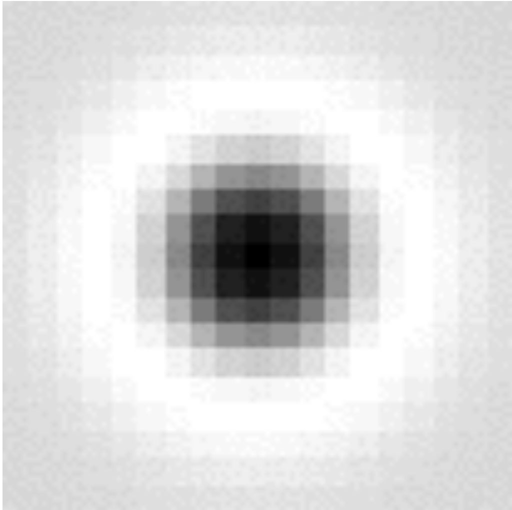

Original
Laplacian
Sobel X
Sobel Y

A bit of mid-term review

Questions: What is Difference of Gaussians? Laplacian kernel? What are their relationships? Can you plot the functions?

- The *Laplacian of Gaussian* (*LoG*)



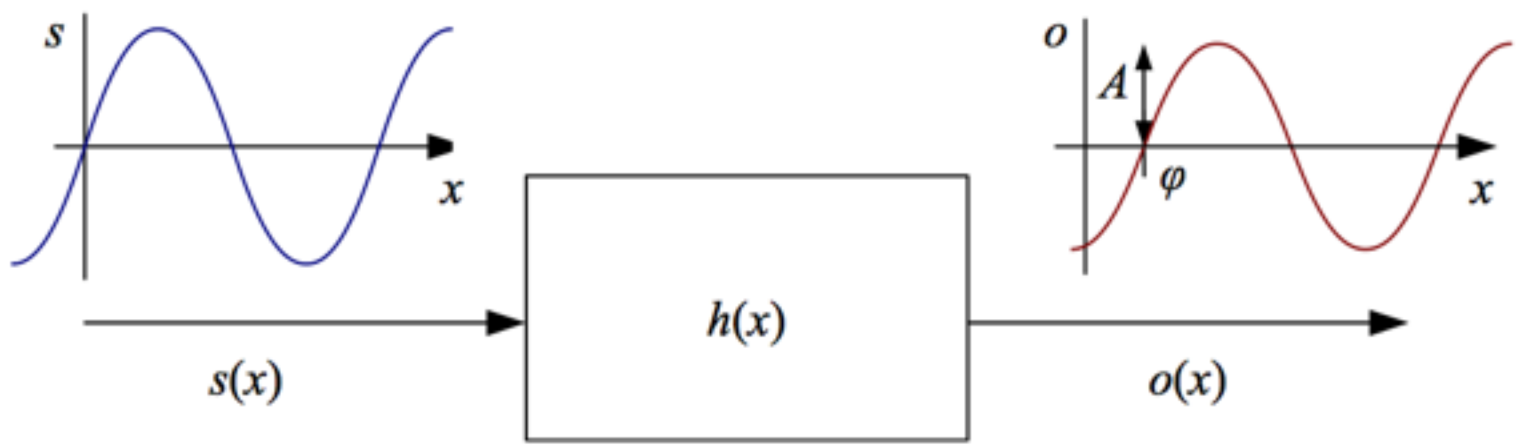


$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

(very similar to a Difference of Gaussians (DoG) – i.e. a Gaussian minus a slightly smaller Gaussian)

A bit of mid-term review

Fourier Transform of a sinusoid is a sinusoid



Questions: What are Fourier transform of Gaussian, Box Filter, Laplacian functions?

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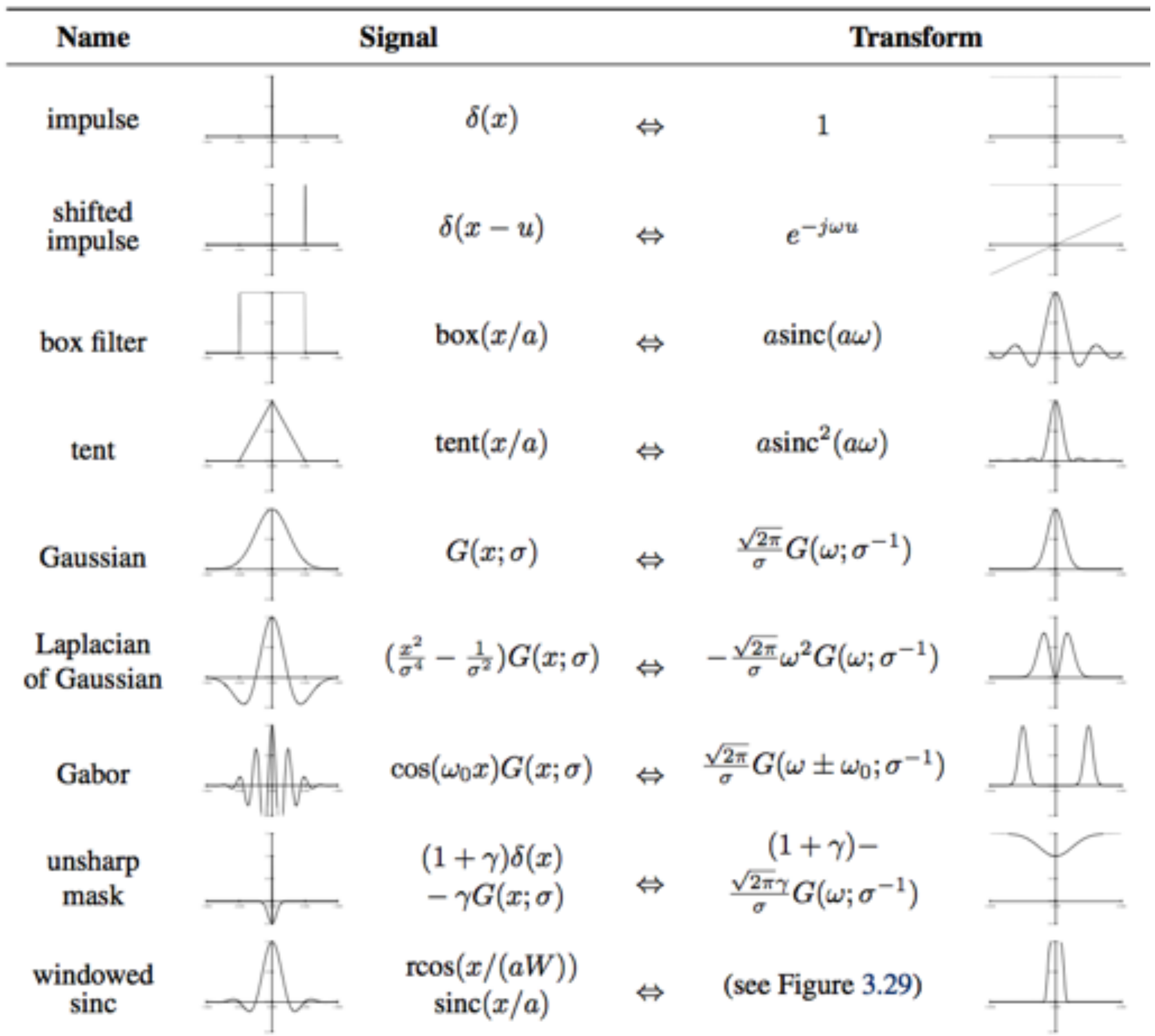
A bit of mid-term review



Name	Signal		Transform
impulse	$\delta(x)$	$\Leftrightarrow$	1
shifted impulse	$\delta(x-u)$	$\Leftrightarrow$	$e^{-j\omega u}$
box filter	$\text{box}(x/a)$	$\Leftrightarrow$	$a\text{sinc}(a\omega)$
tent	$\text{tent}(x/a)$	$\Leftrightarrow$	$a\text{sinc}^2(a\omega)$
Gaussian	$G(x;\sigma)$	$\Leftrightarrow$	$\frac{\sqrt{2\pi}}{\sigma}G(\omega;\sigma^{-1})$
Laplacian of Gaussian	$(\frac{x^2}{\sigma^4}-\frac{1}{\sigma^2})G(x;\sigma)$	$\Leftrightarrow$	$-\frac{\sqrt{2\pi}}{\sigma}\omega^2 G(\omega;\sigma^{-1})$
Gabor	$\cos(\omega_0 x)G(x;\sigma)$	$\Leftrightarrow$	$\frac{\sqrt{2\pi}}{\sigma}G(\omega\pm\omega_0;\sigma^{-1})$
unsharp mask	$(1+\gamma)\delta(x)$ $-\gamma G(x;\sigma)$	$\Leftrightarrow$	$(1+\gamma)-$ $\frac{\sqrt{2\pi}\gamma}{\sigma}G(\omega;\sigma^{-1})$
windowed sinc	$\text{rcos}(x/(aW))$ $\text{sinc}(x/a)$	$\Leftrightarrow$	(see Figure 3.29)

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A bit of mid-term review

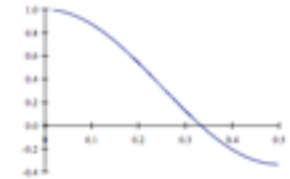
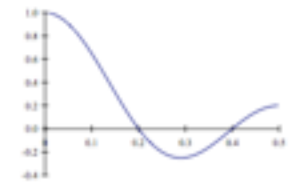
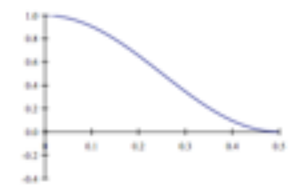
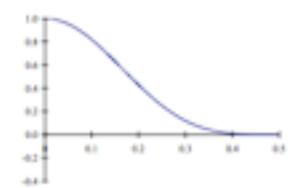
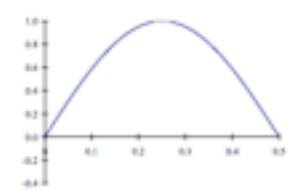
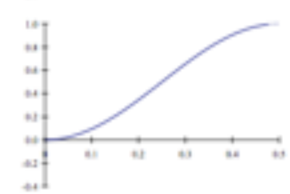
Name	Kernel	Transform	Plot
box-3	$\frac{1}{3}\begin{bmatrix}1 & 1 & 1\end{bmatrix}$	$\frac{1}{3}(1+2\cos\omega)$	
box-5	$\frac{1}{5}\begin{bmatrix}1 & 1 & 1 & 1 & 1\end{bmatrix}$	$\frac{1}{5}(1+2\cos\omega+2\cos 2\omega)$	
linear	$\frac{1}{4}\begin{bmatrix}1 & 2 & 1\end{bmatrix}$	$\frac{1}{2}(1+\cos\omega)$	
binomial	$\frac{1}{16}\begin{bmatrix}1 & 4 & 6 & 4 & 1\end{bmatrix}$	$\frac{1}{4}(1+\cos\omega)^2$	
Sobel	$\frac{1}{2}\begin{bmatrix}-1 & 0 & 1\end{bmatrix}$	$\sin\omega$	
corner	$\frac{1}{2}\begin{bmatrix}-1 & 2 & -1\end{bmatrix}$	$\frac{1}{2}(1-\cos\omega)$	

Table 3.3 Fourier transforms of the separable kernels shown in Figure 3.14.

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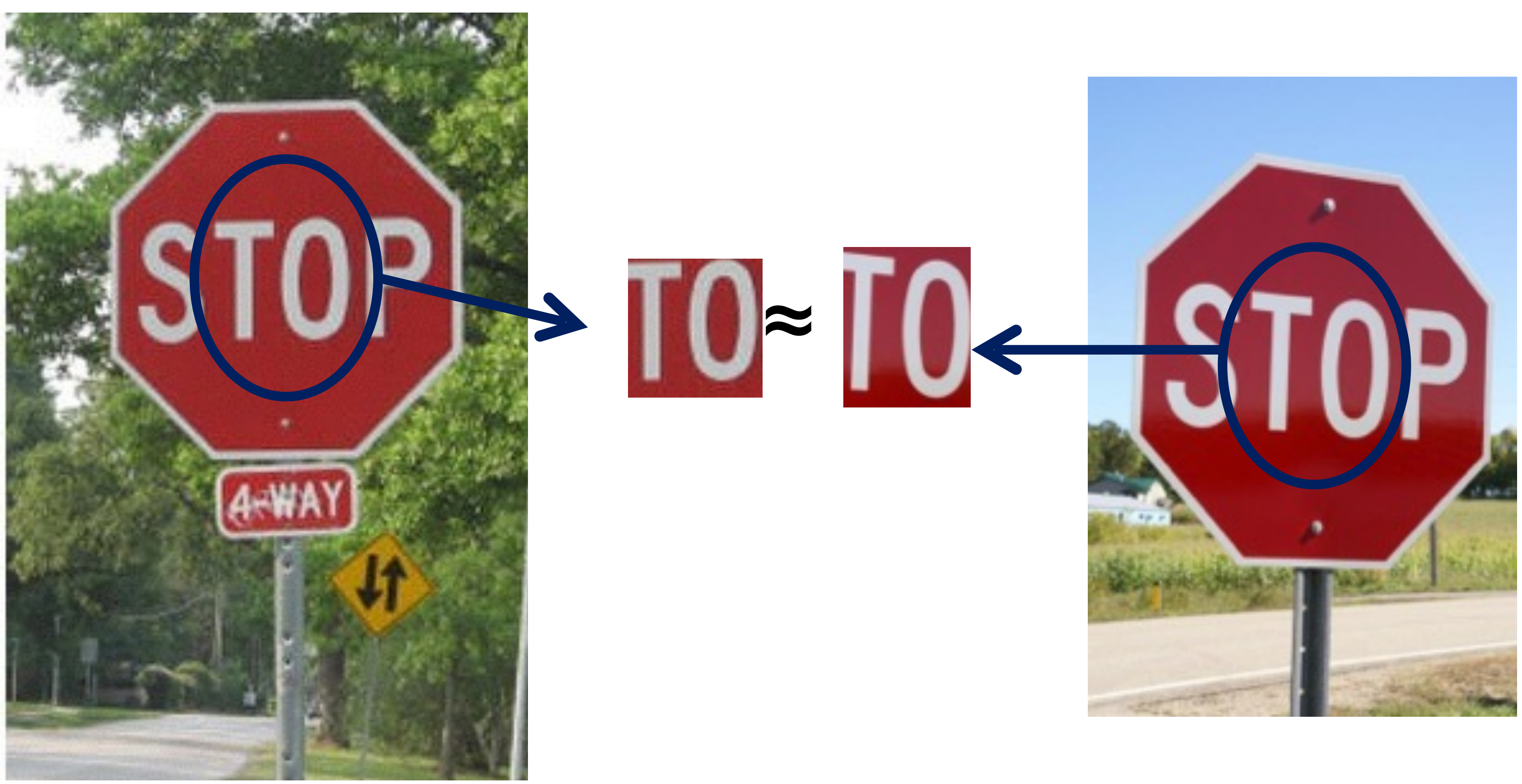
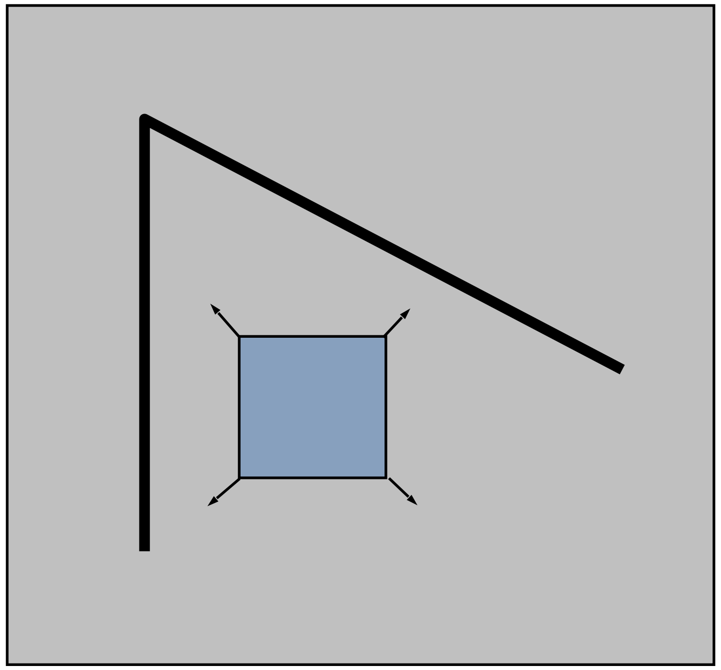
This section: correspondence and alignment

- Correspondence: matching points, patches, edges, or regions across images

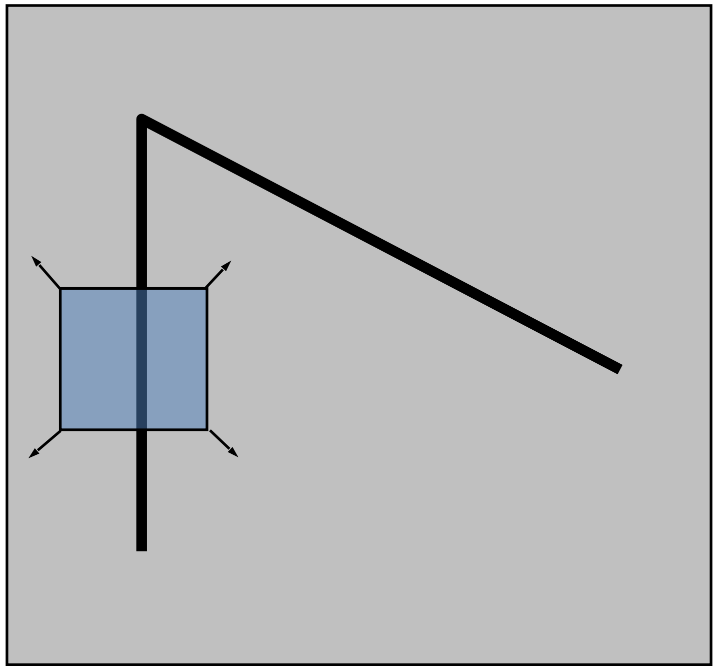


Local measure of feature uniqueness

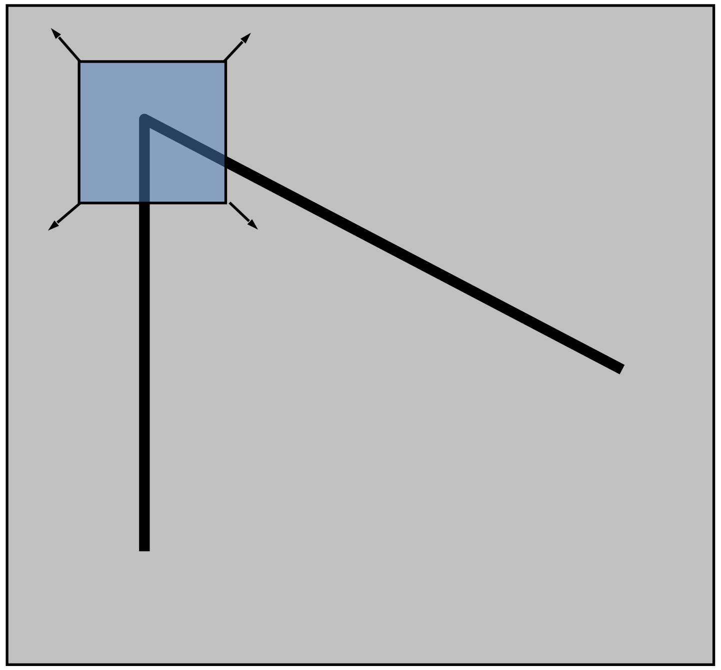
- How does the window change when you shift it?
- Shifting the window in any direction causes a big change



"flat" region:
no change in all directions



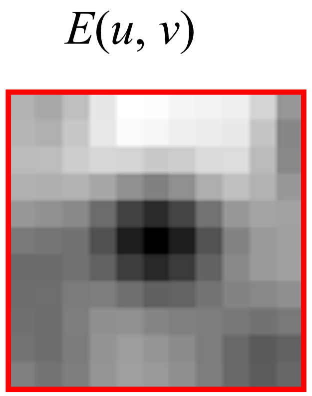
"edge":
no change along the edge direction

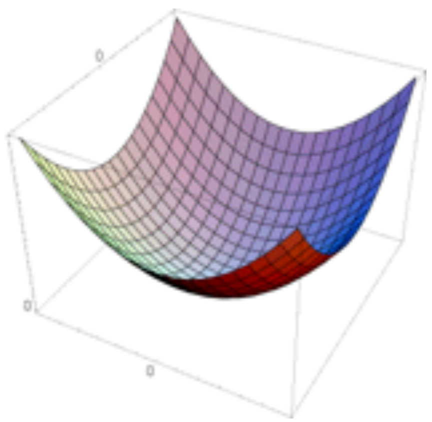


"corner":
significant change in all directions

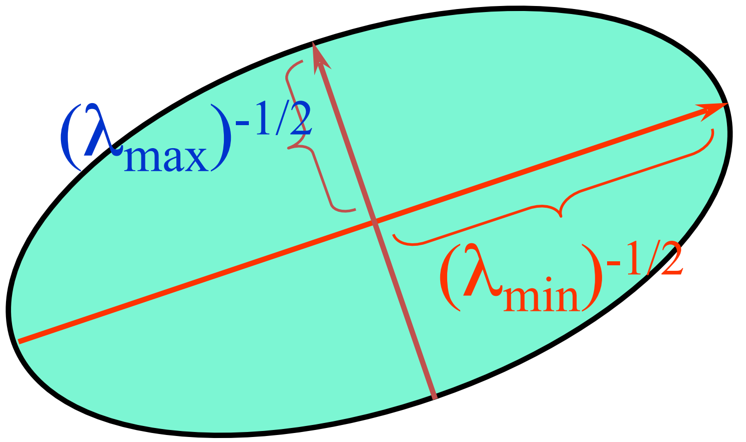
Credit: S. Seitz, D. Frolova, D. Simakov

Review: Harris corner detector





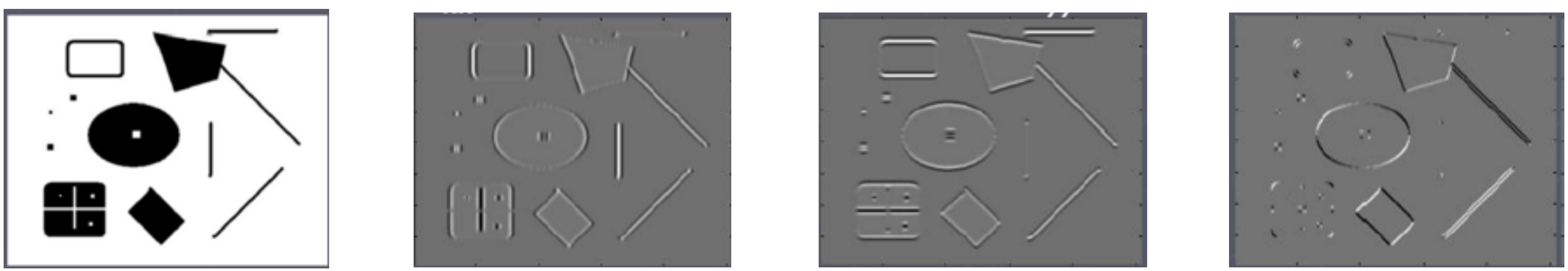
- Approximate distinctiveness by local auto-correlation.
- Approximate local auto-correlation by second moment matrix
- Quantify distinctiveness (or cornerness) as function of the eigenvalues of the second moment matrix.
- But we don't actually need to compute the eigenvalues by using the determinant and trace of the second moment matrix.



Corners as distinctive interest points

$$M = \sum \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

2 x 2 matrix of image derivatives (averaged in neighborhood of a point)



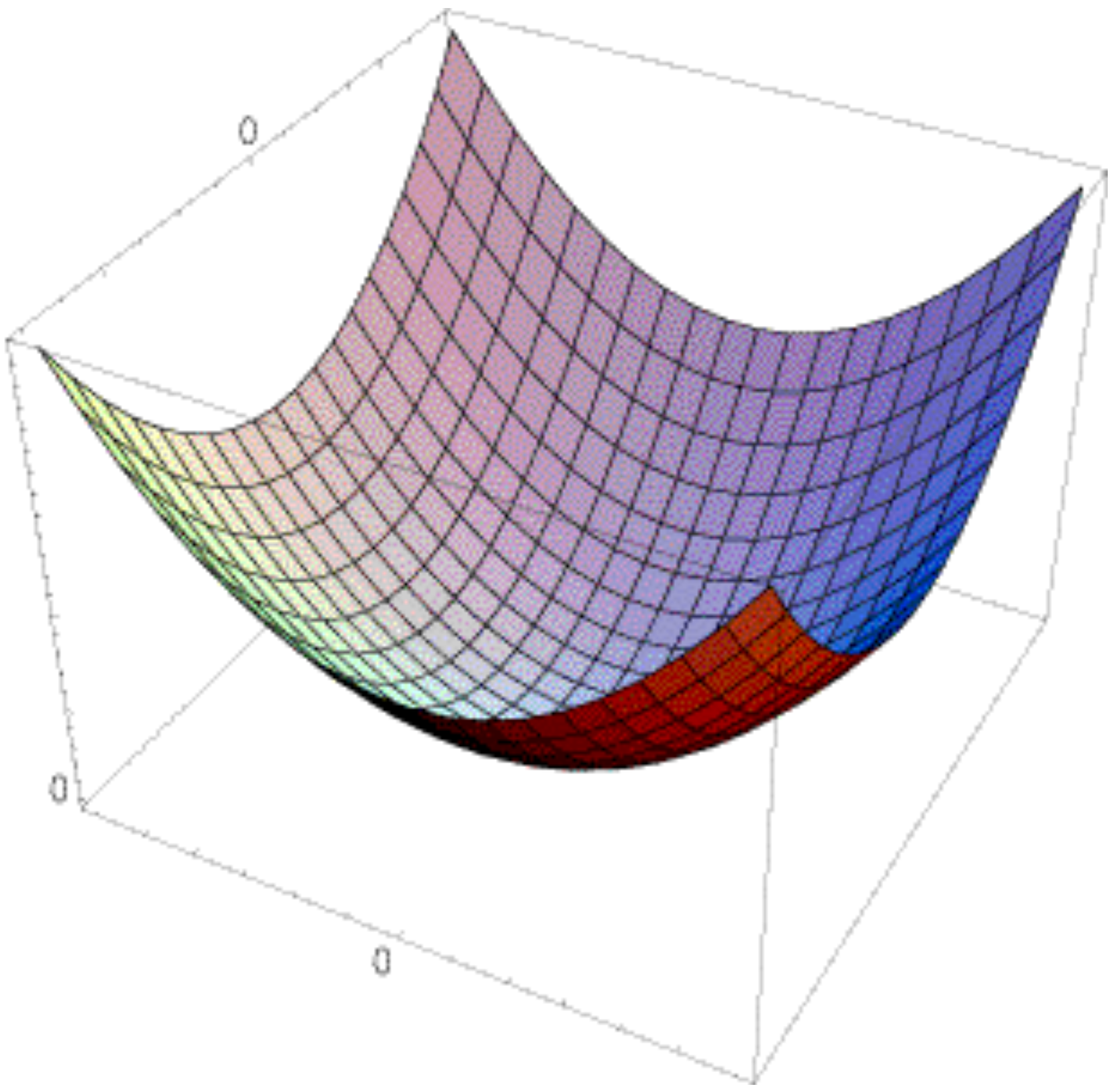
Notation: $I_x \Leftrightarrow \frac{\partial I}{\partial x}$ $\quad I_y \Leftrightarrow \frac{\partial I}{\partial y}$ $\quad I_x I_y \Leftrightarrow \frac{\partial I}{\partial x}\frac{\partial I}{\partial y}$

Interpreting the second moment matrix

The surface $E(u,v)$ is locally approximated by a quadratic form. Let's try to understand its shape.

$$E(u,v) \approx [u \ \ v] \ M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Quick eigenvalue/eigenvector review

The **eigenvectors** of a matrix **A** are the vectors **x** that satisfy:

$$Ax = \lambda x$$

The scalar λ is the **eigenvalue** corresponding to **x**

– The eigenvalues are found by solving:

$$det(A - \lambda I) = 0$$

– In our case, ***A*** = ***H*** is a 2x2 matrix, so we have

$$det \begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix} = 0$$

– The solution:

$$\lambda_{\pm} = \frac{1}{2}\left[(h_{11} + h_{22}) \pm \sqrt{4h_{12}h_{21} + (h_{11} - h_{22})^2}\right]$$

Once you kno

$$\begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Quick eigenvalue/eigenvector review

- The solution:

$$\lambda_{\pm} = \frac{1}{2}\left[(h_{11} + h_{22}) \pm \sqrt{4h_{12}h_{21} + (h_{11} - h_{22})^2}\right]$$

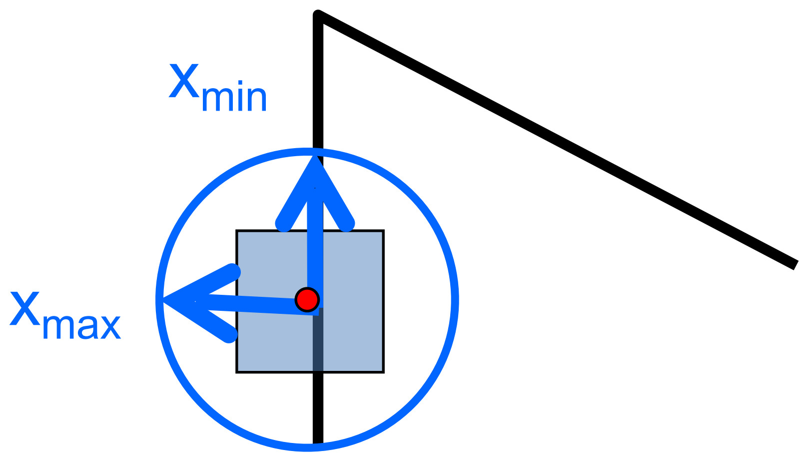
Once you know λ, you find the eigenvectors by solving

$$\begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Symmetric, square matrix: eigenvectors are mutually orthogonal

Corner detection: the math

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{H} \begin{bmatrix} u \\ v \end{bmatrix}$$



$$Hx_{\max} = \lambda_{\max} x_{\max}$$

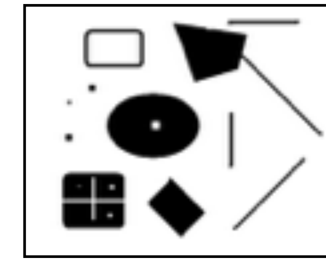
$$Hx_{\min} = \lambda_{\min} x_{\min}$$

Eigenvalues and eigenvectors of H

- Define shift directions with the smallest and largest change in error
- x_{max} = direction of largest increase in *E*
- λ_{max} = amount of increase in direction x_{max}
- x_{min} = direction of smallest increase in *E*
- λ_{min} = amount of increase in direction x_{min}

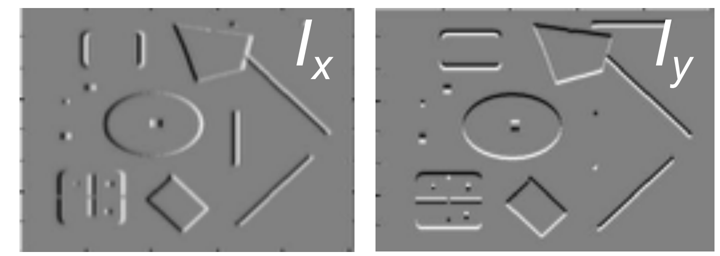
Harris Detector [Harris88]

- Second moment matrix



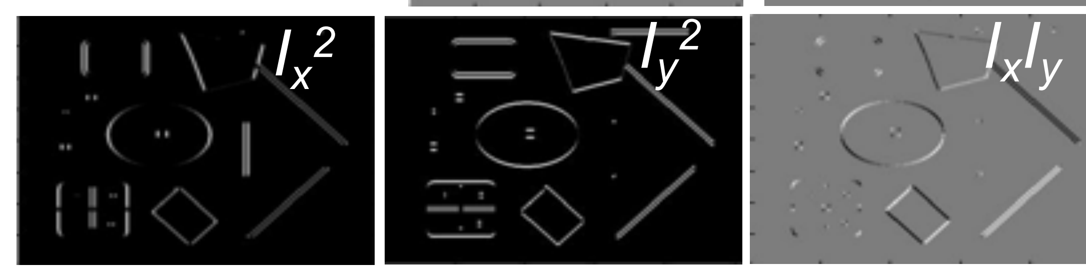
$$\mu(\sigma_I,\sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

1. Image derivatives (optionally, blur first)

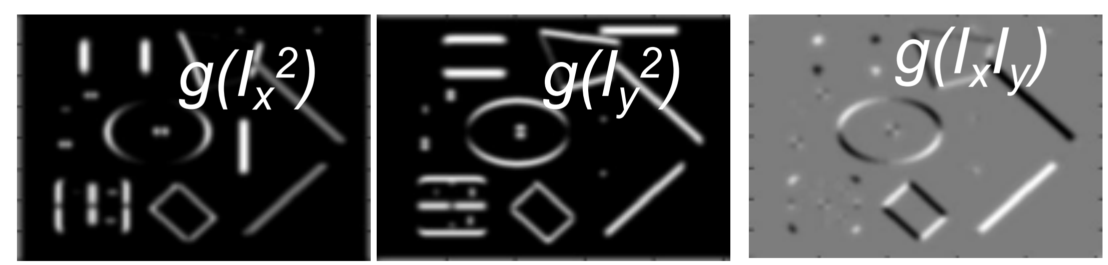


$$\det M = \lambda_1 \lambda_2$$
$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

2. Square of derivatives



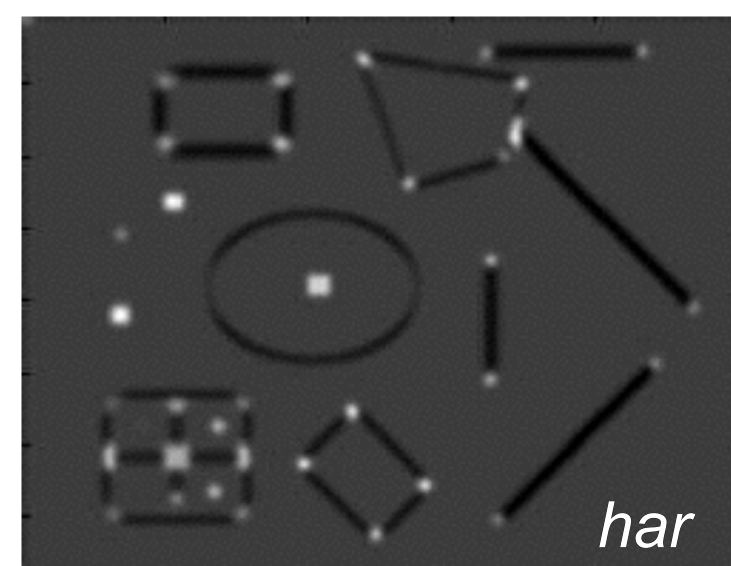
3. Gaussian filter $g(\sigma_I)$



4. Cornerness function – both eigenvalues are strong

$$har = \det[\mu(\sigma_I,\sigma_D)] - \alpha[\operatorname{trace}(\mu(\sigma_I,\sigma_D))^2] =$$
$$g(I_x^2)g(I_y^2) - [g(I_x I_y)]^2 - \alpha[g(I_x^2) + g(I_y^2)]^2$$

5. Non-maxima suppression



So far: can localize in x-y, but not scale

Scale Space

Scale-space representation $L(x, y; t)$ at scale $t = 0$, corresponding to the original image f

Scale-space representation $L(x, y; t)$ at scale $t = 1$

Scale-space representation $L(x, y; t)$ at scale $t = 4$

Scale-space representation $L(x, y; t)$ at scale $t = 16$

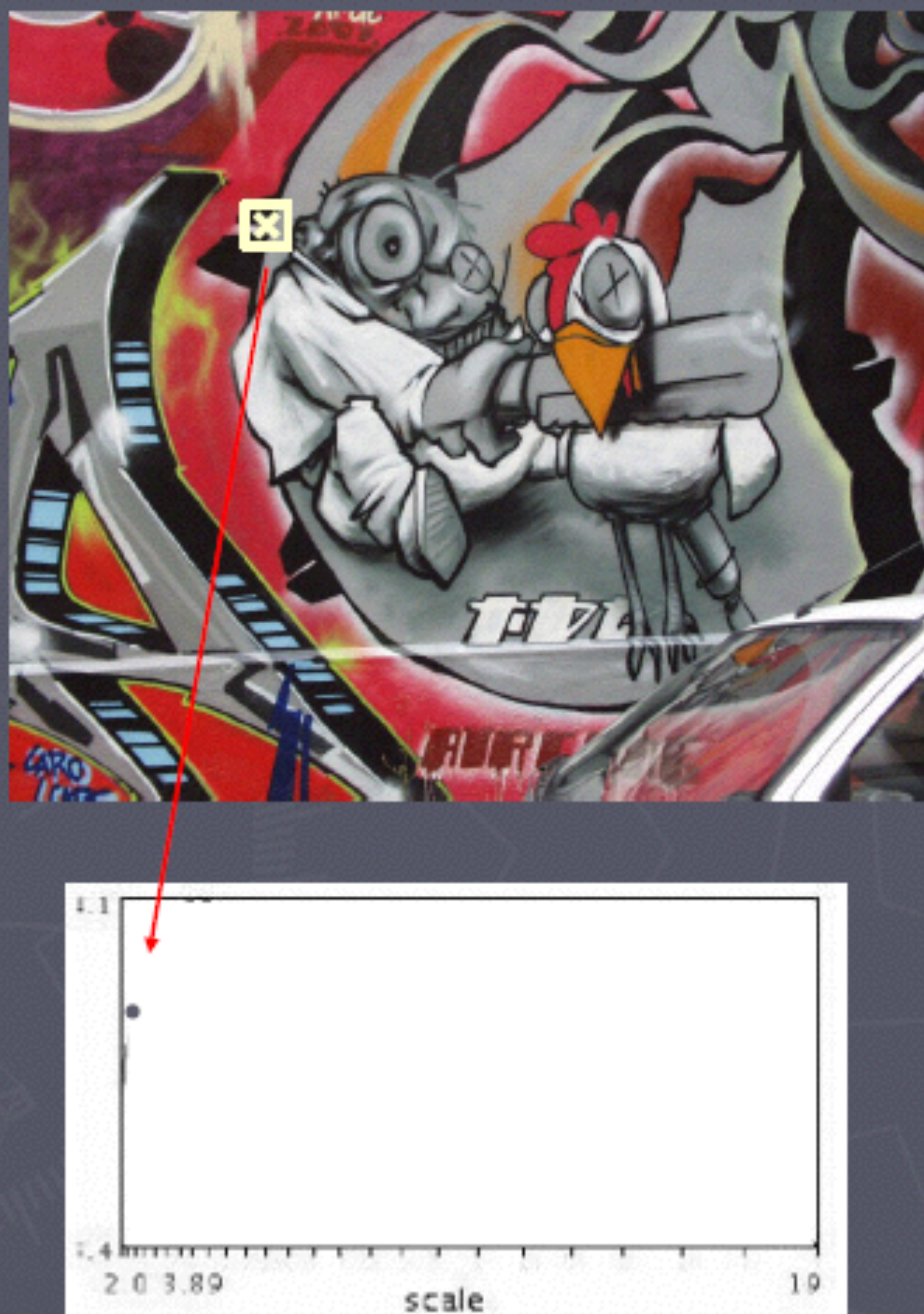
For an given image f(x,y)

$$g(x, y; t) = \frac{1}{2\pi t} e^{-(x^2+y^2)/2t}$$

$$L(\cdot, \cdot; t) = g(\cdot, \cdot; t) * f(\cdot, \cdot)$$

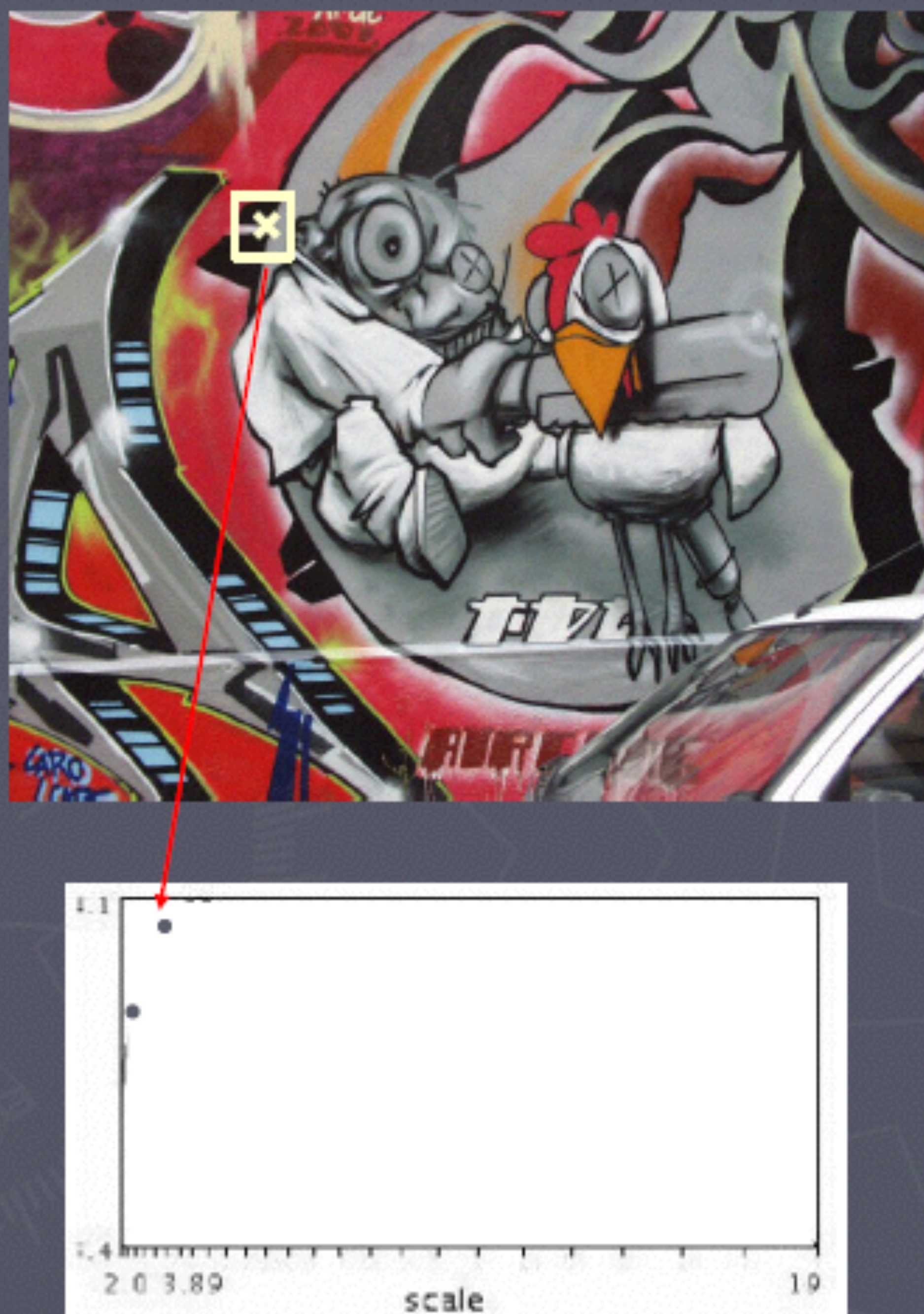
Automatic scale selection

Lindeberg et al., 1996



$f(I_{i_1 \ldots i_m}(x,\sigma))$

Automatic scale selection



$f(I_{i_1 \ldots i_m}(x, \sigma))$

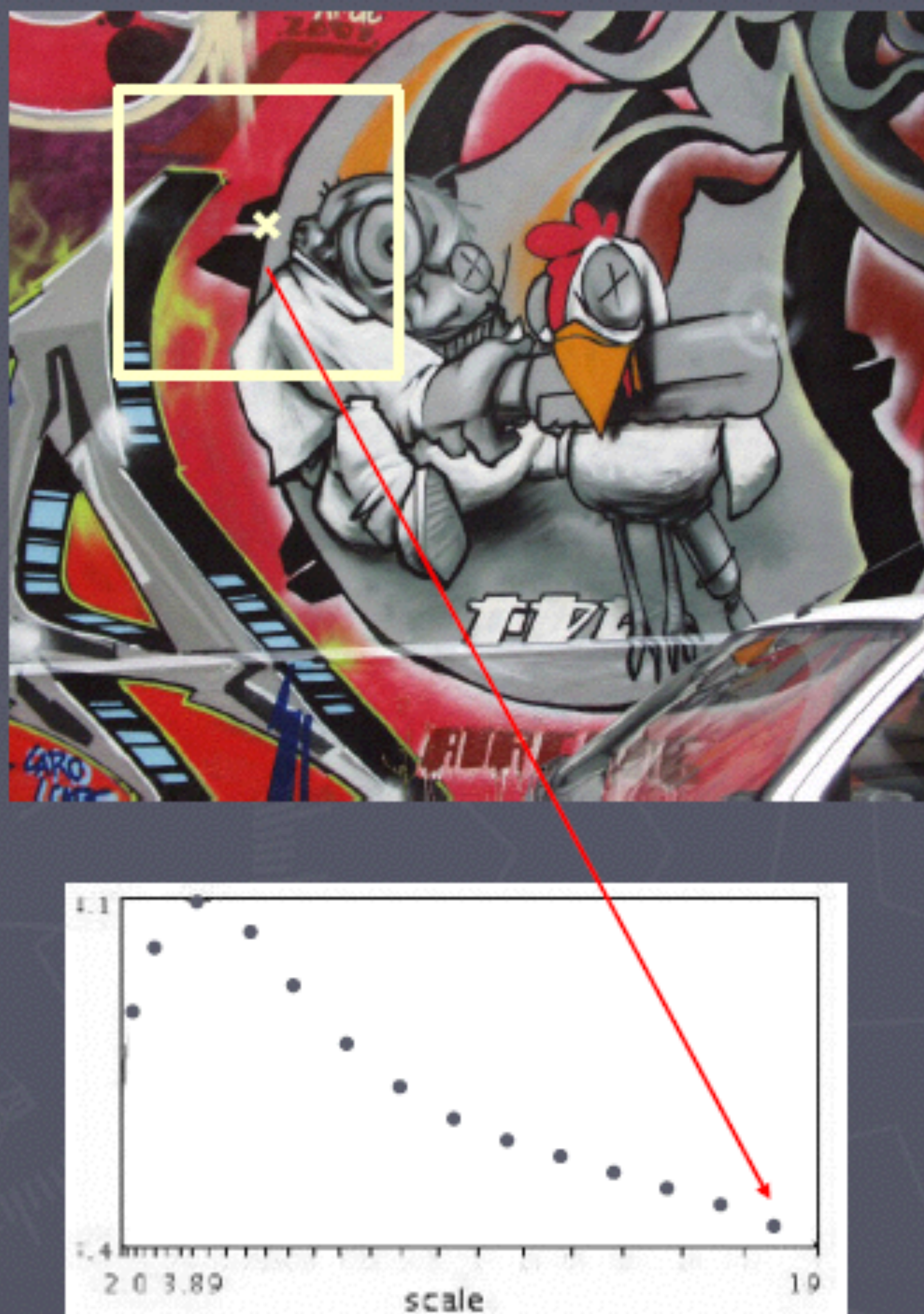
Automatic scale selection

$$f(I_{i_1 \ldots i_m}(x,\sigma))$$

Automatic scale selection

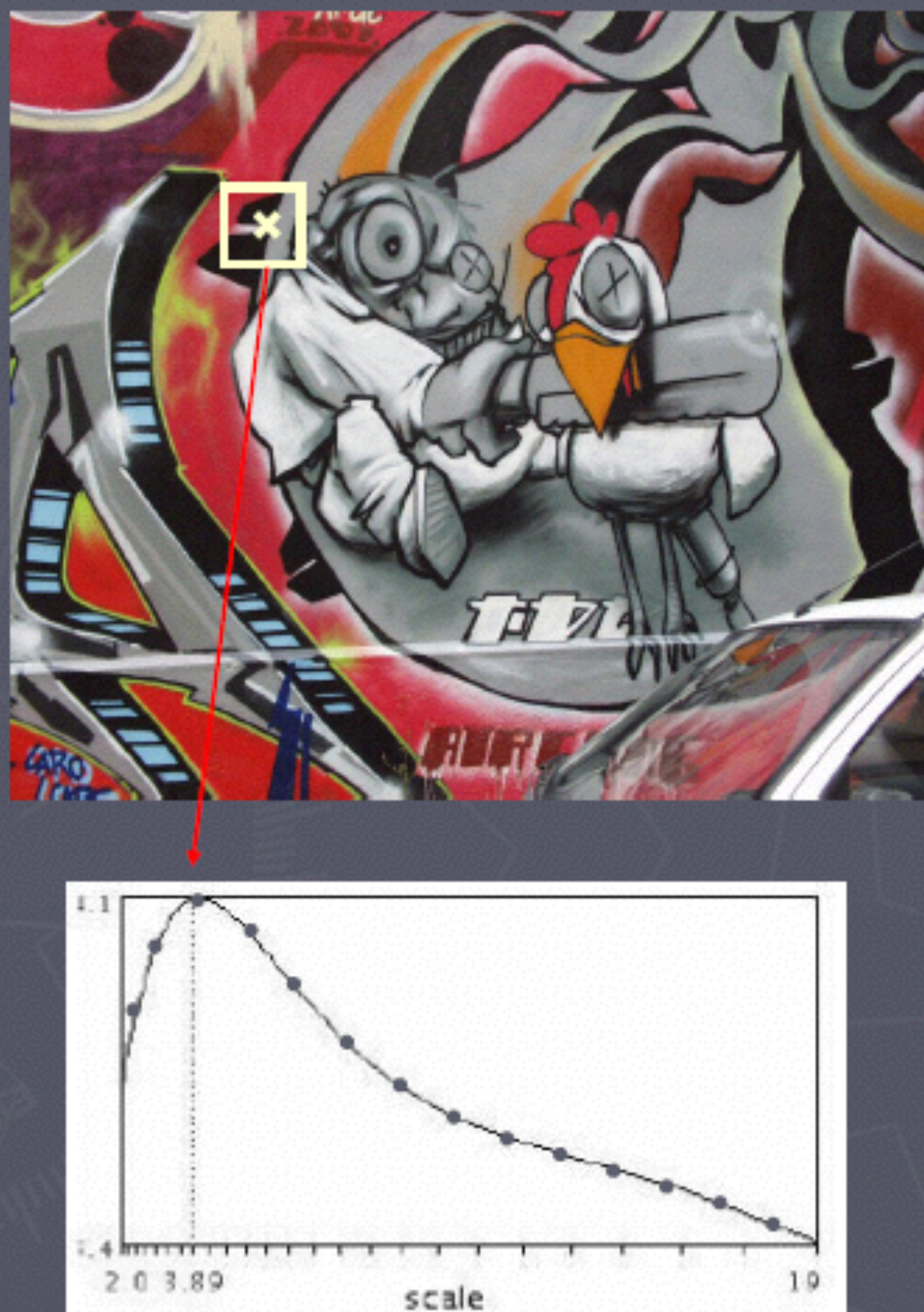
$$f(I_{i_1 \ldots i_m}(x,\sigma))$$

Automatic scale selection



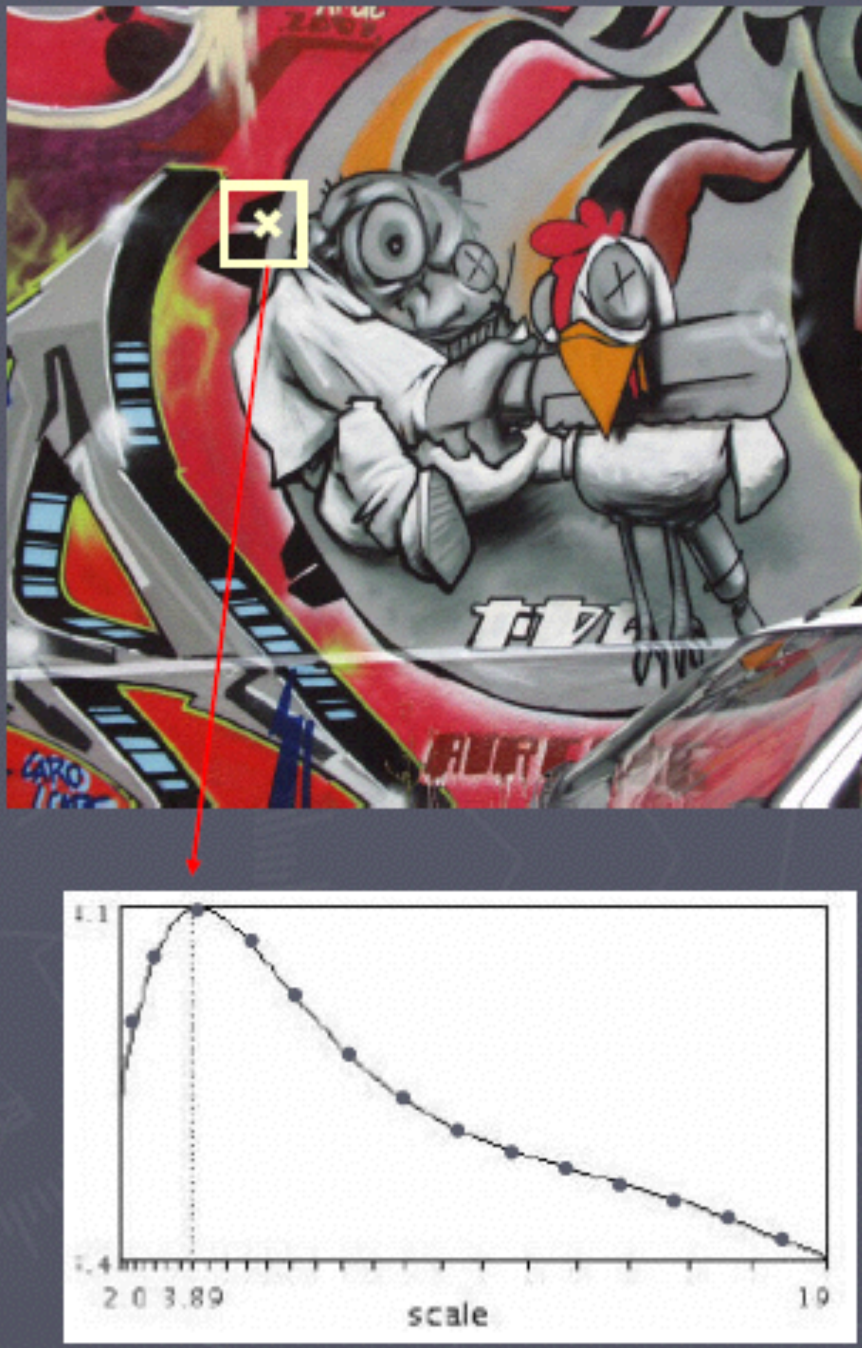
$f(I_{i_1 \ldots i_m}(x,\sigma))$

Automatic scale selection



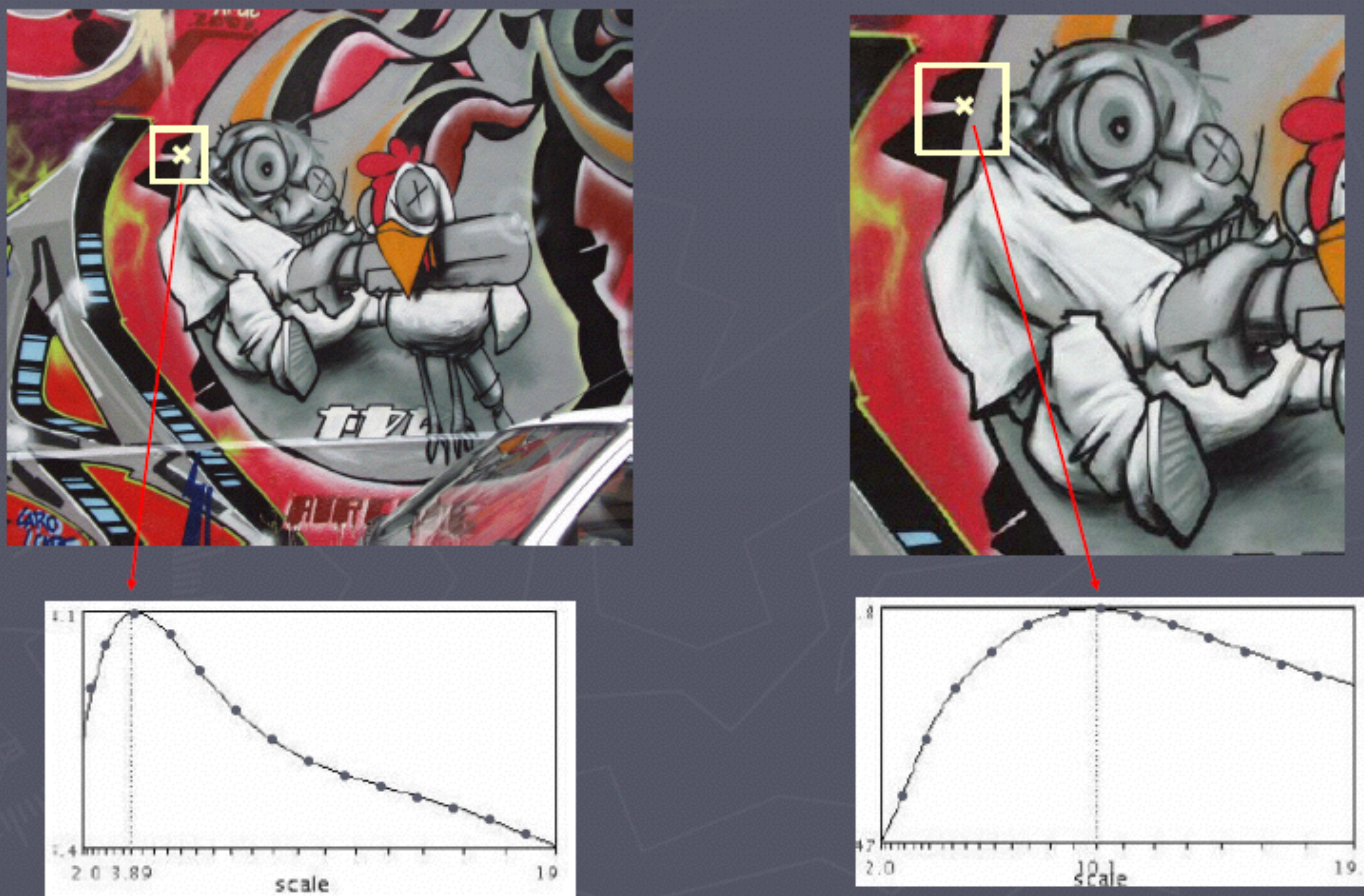
$$f(I_{i_1 \dots i_m}(x,\sigma))$$

Automatic scale selection



$f(I_{i_1 \ldots i_m}(x,\sigma))$

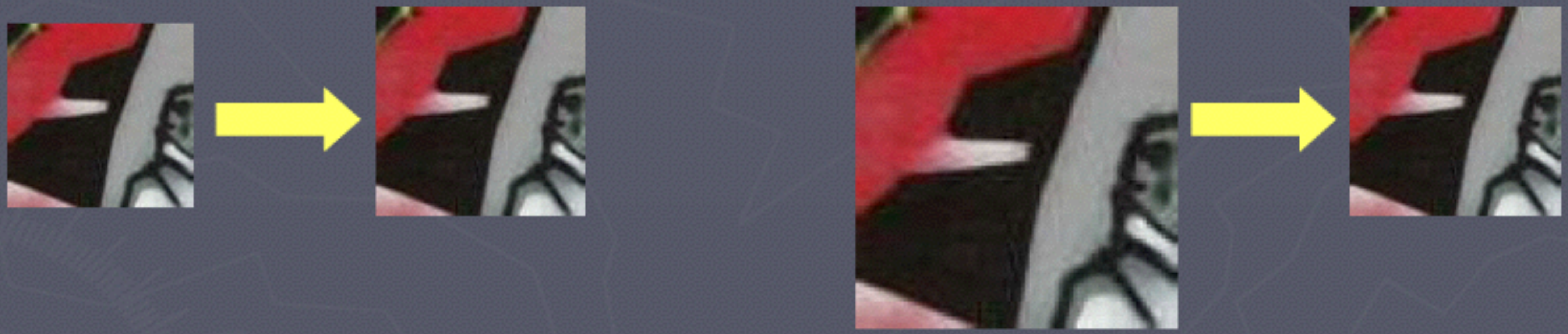
Automatic scale selection



$f(I_{i_1 \dots i_m}(x, \sigma))$ $f(I_{i_1 \dots i_m}(x', \sigma'))$

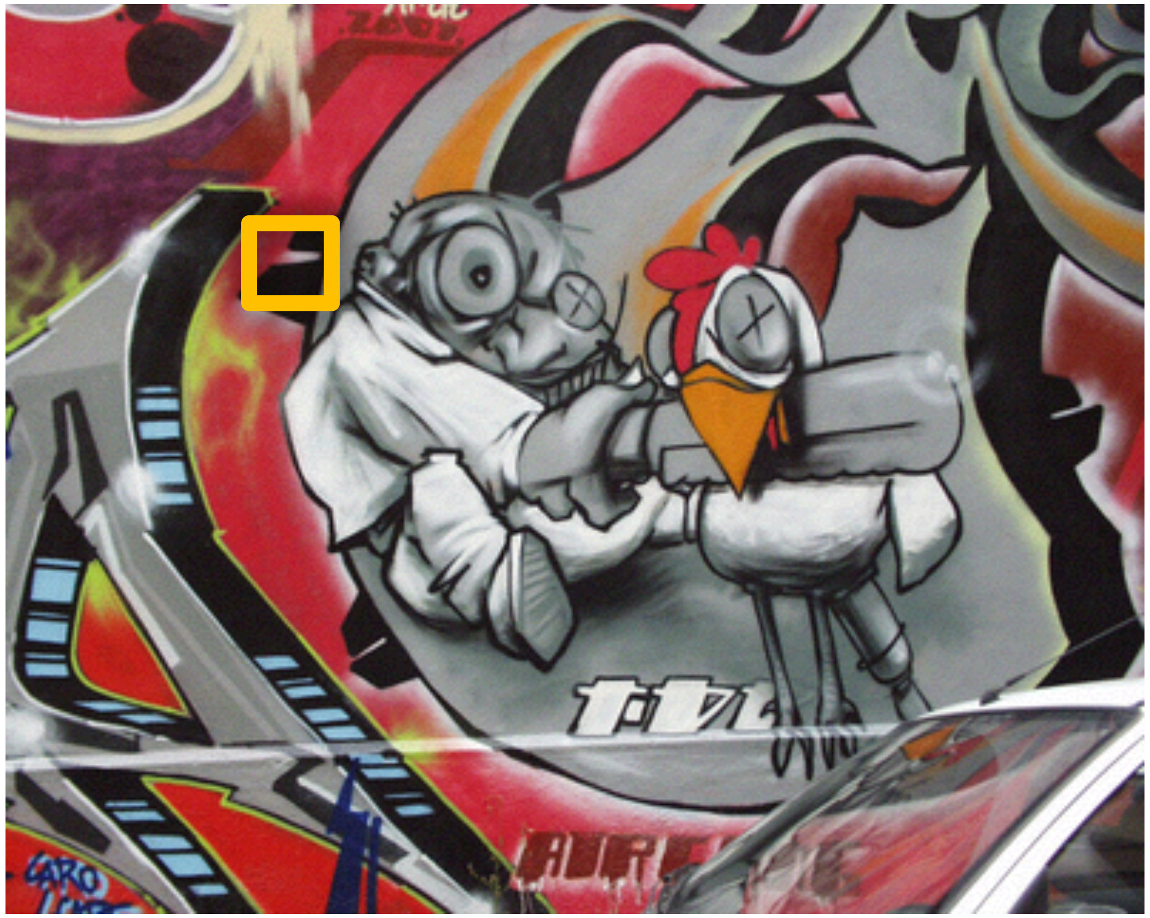
Automatic scale selection

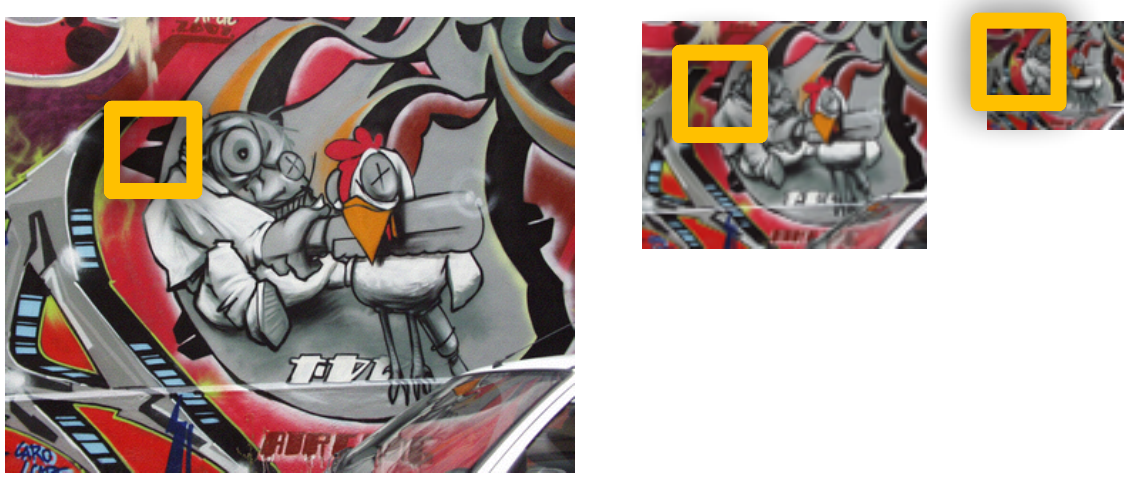
Normalize: rescale to fixed size



Implementation

- Instead of computing *f* for larger and larger windows, we can implement using a fixed window size with a Gaussian pyramid

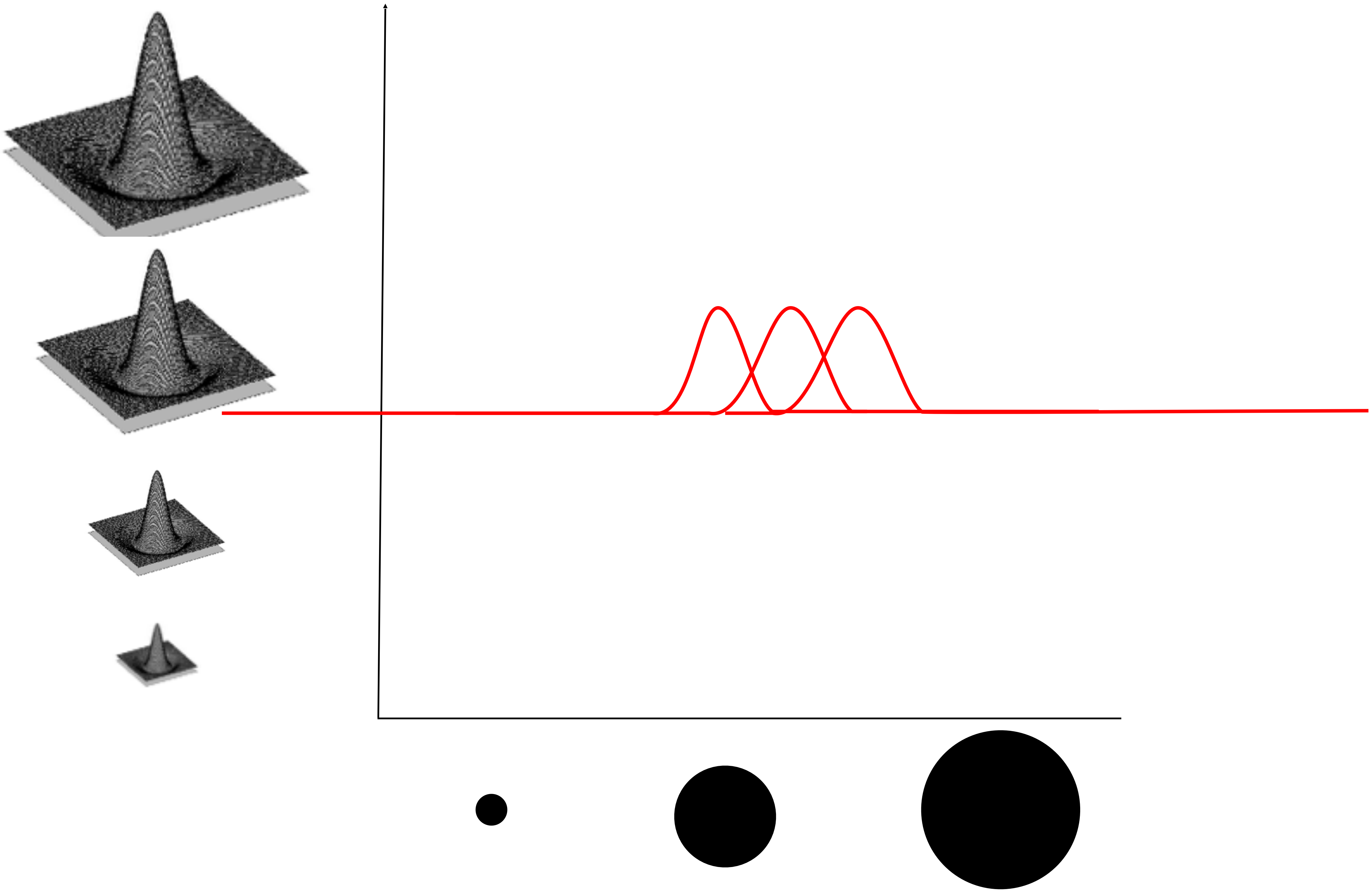




(sometimes need to create in-between levels, e.g. a ¾-size image)

What Is A Useful Signature Function?

- Difference-of-Gaussian = “blob” detector



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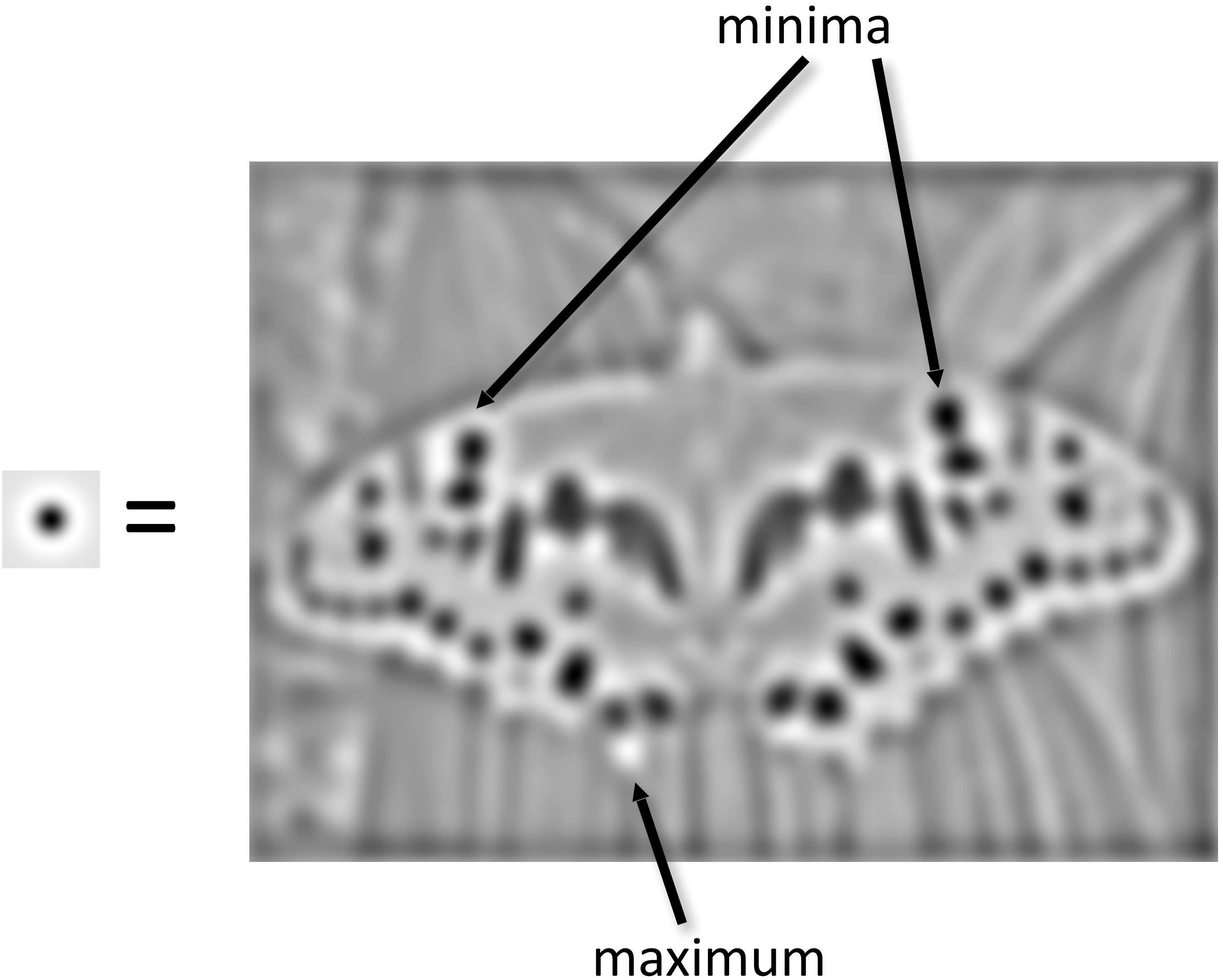
Laplacian of Gaussian

- "Blob" detector

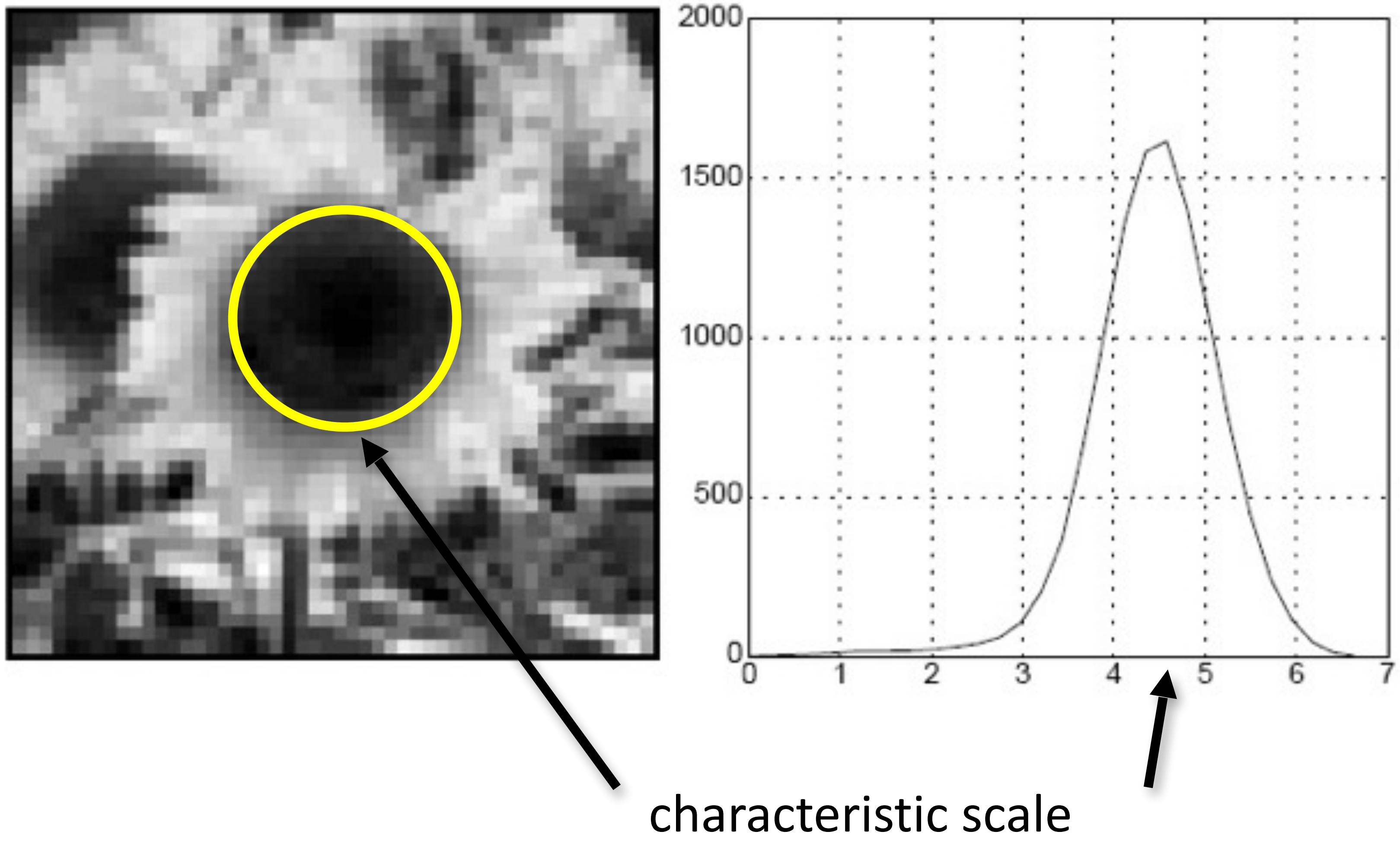
*



- Find maxima *and minima* of LoG operator in space and scale

Characteristic scale

- We define the characteristic scale as the scale that produces peak of Laplacian response



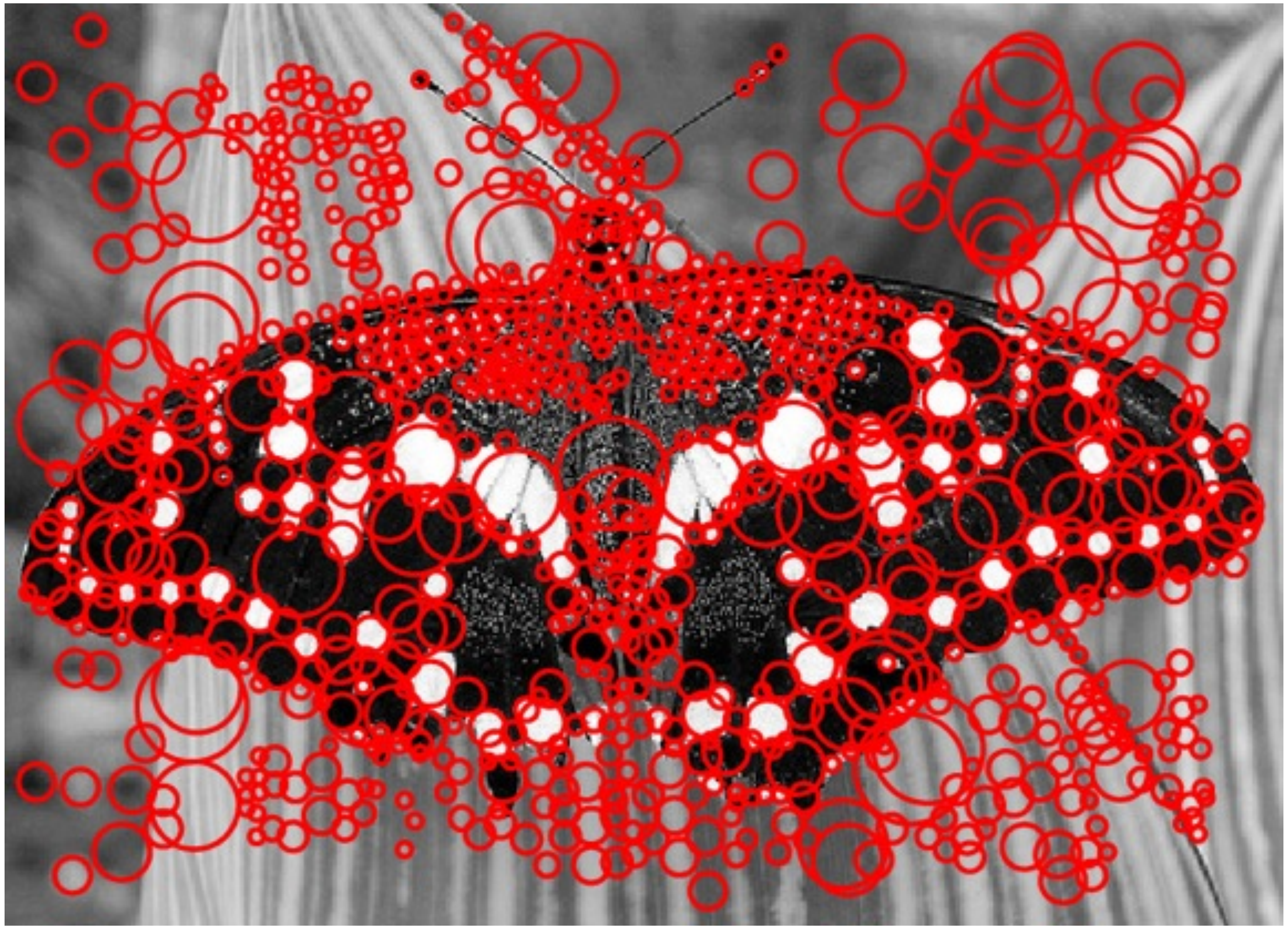
T. Lindeberg (1998). ["Feature detection with automatic scale selection."]() *International Journal of Computer Vision* **30** (2): pp 77--116.

Scale-space blob detector: Example

Scale-space blob detector: Example

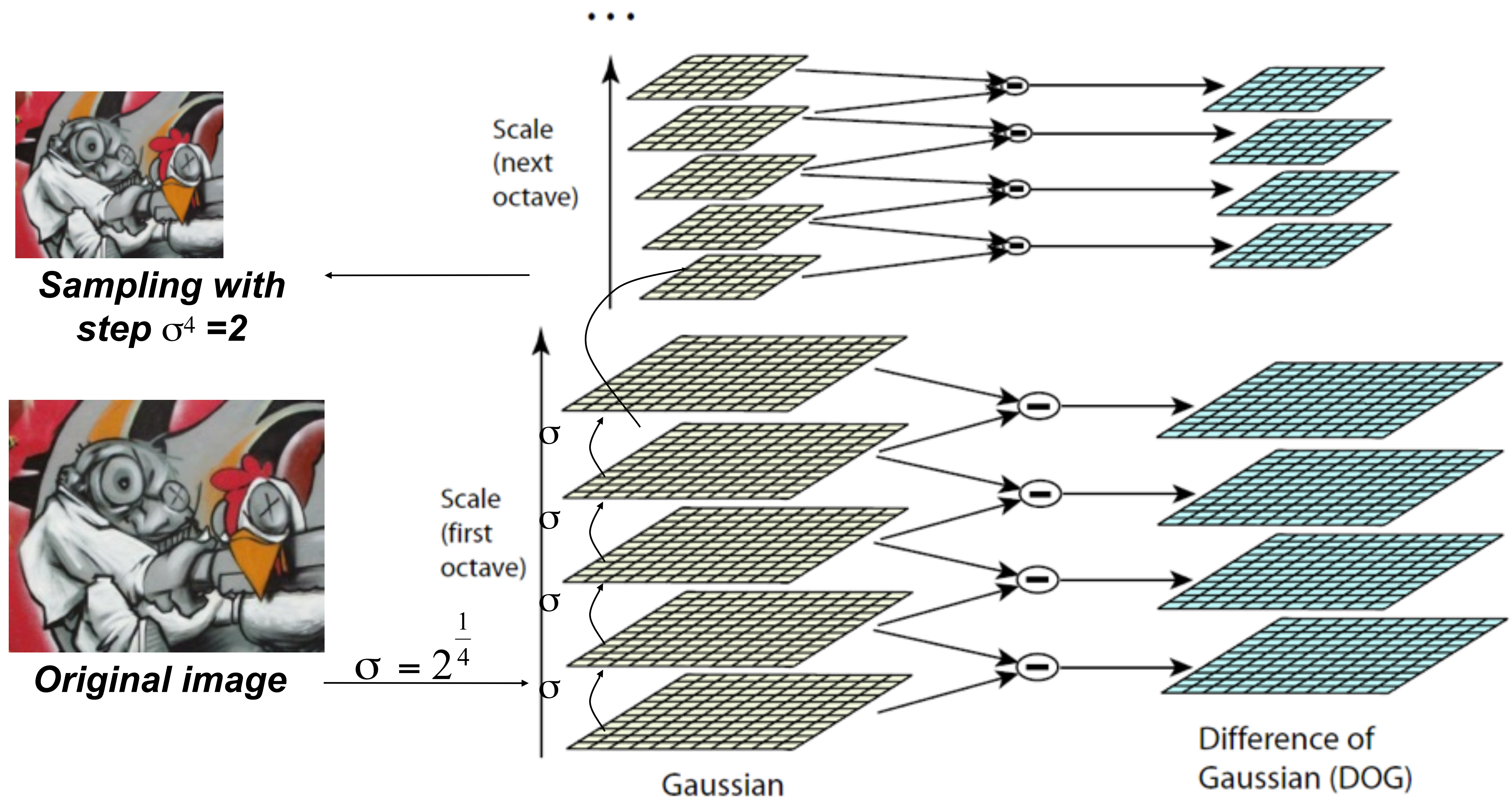
sigma = 11.9912

Scale-space blob detector: Example



DoG – Efficient Computation

- Computation in Gaussian scale pyramid



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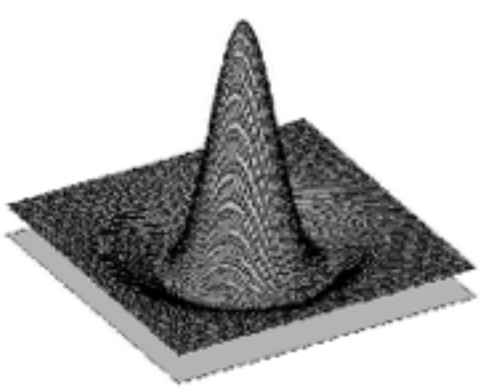
Find local maxima in position-scale space of Difference-of-Gaussian



$L_{xx}(\sigma) + L_{yy}(\sigma)$

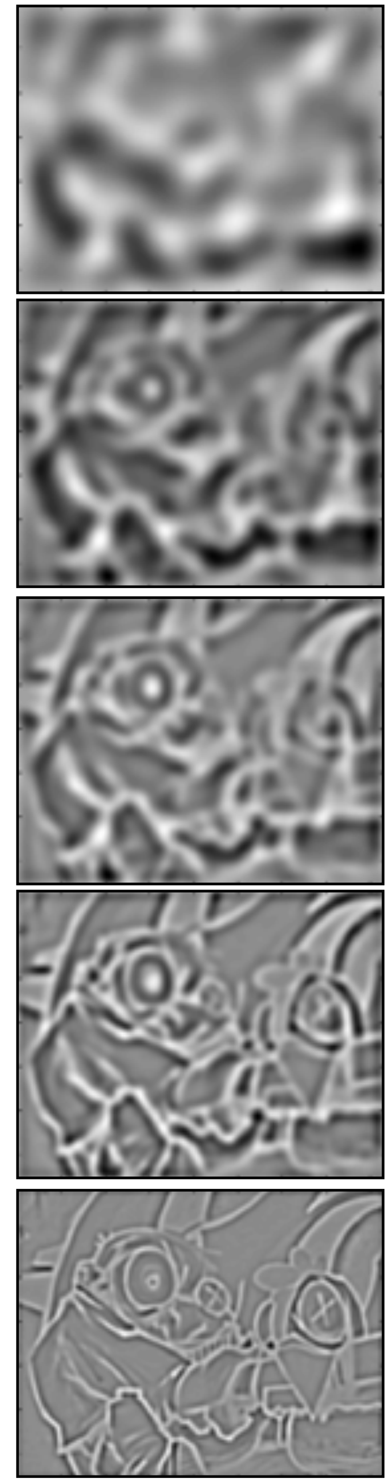
σ_5

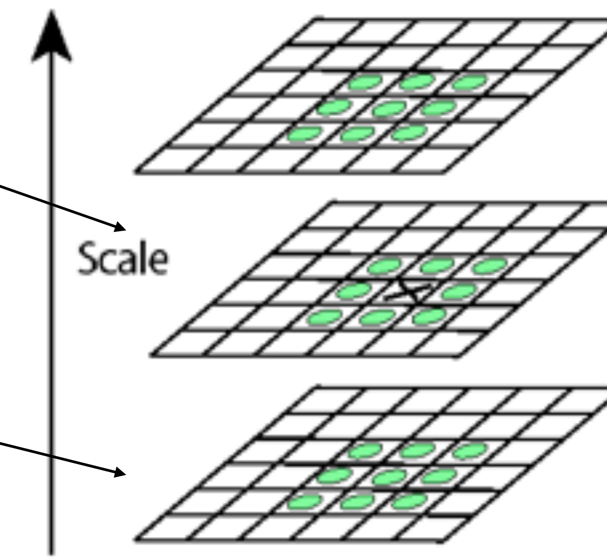
σ_4

σ_3

σ_2

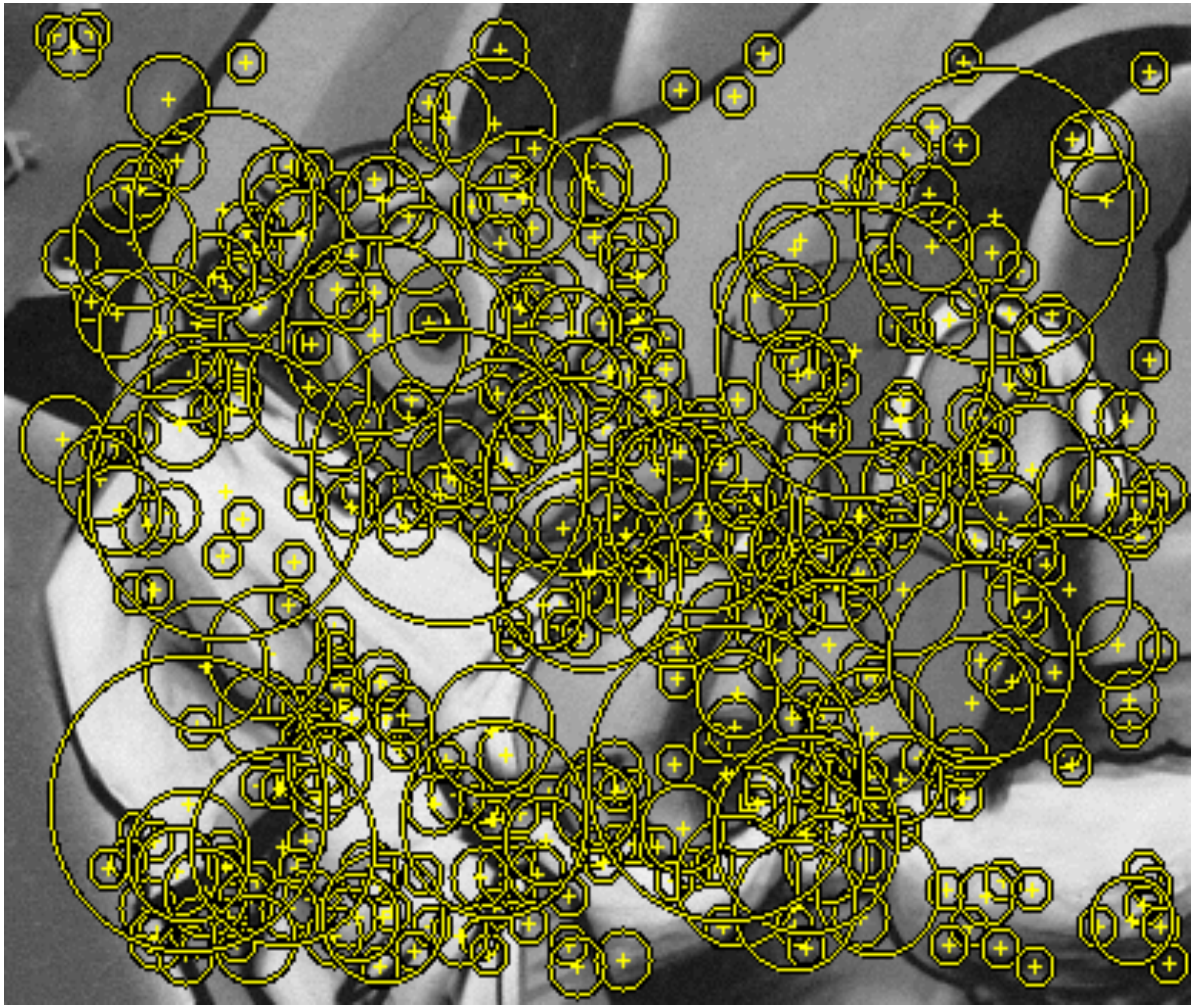
σ





⇒ **List of *(x, y,***

Results: Difference-of-Gaussian



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