Cell Complexes Arising from Bouncing Light Rays

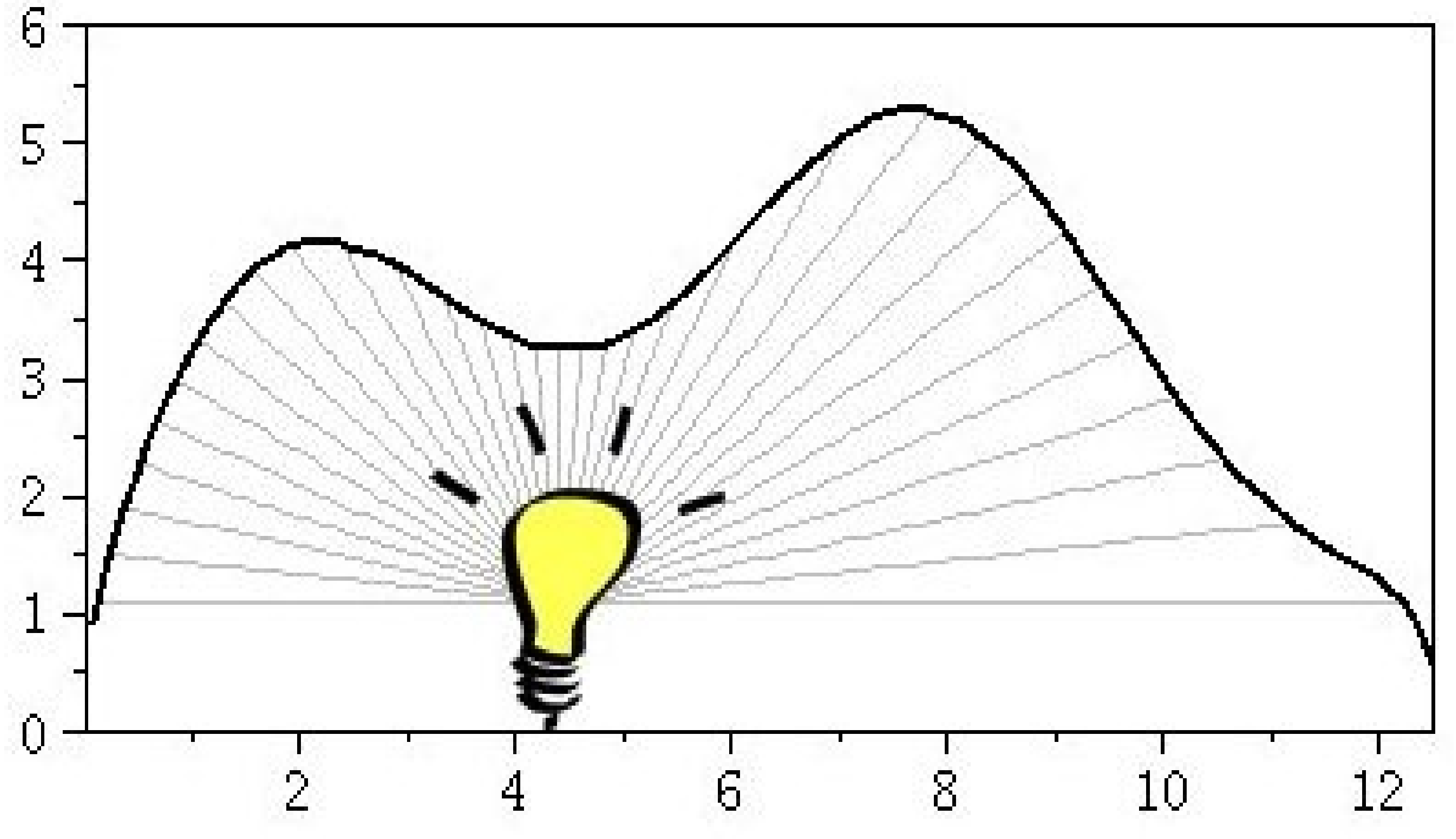
and other fun localization diversions...

Donna A. Dietz
March 20, 2009

explanatory slide

These slides are from a talk given by Donna A. Dietz on March 20, 2009 at Mansfield University at the annual PASSHEMA conference. Donna A. Dietz was at that time on leave of absence from Mansfield University where she was an Associate Professor of Mathematics, and she was at that time also a Visiting Scholar at the University of Pennsylvania. These explanatory slides are to provide the verbal component of the talk.

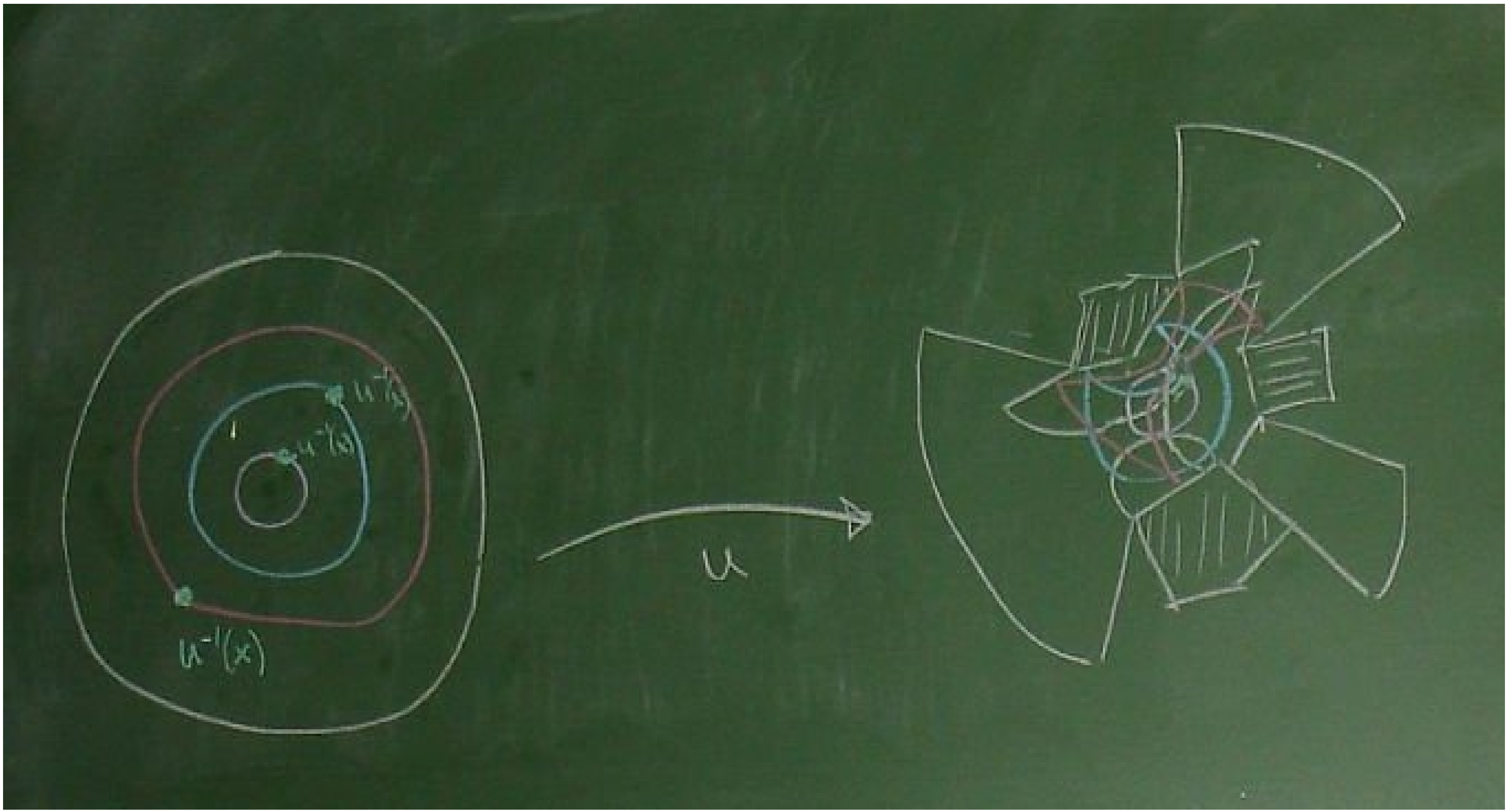
Simple Model



explanatory slide

The problem to be explored comes from the well-known and simple geometric model for light, sound, radio, and other types of radiation. The model to be used is a simple line-of-sight in two dimensions plus a single bounce allowed per ray. The rays are reflected by the walls given in the diagrams and do not pass through. No diffraction will be considered.

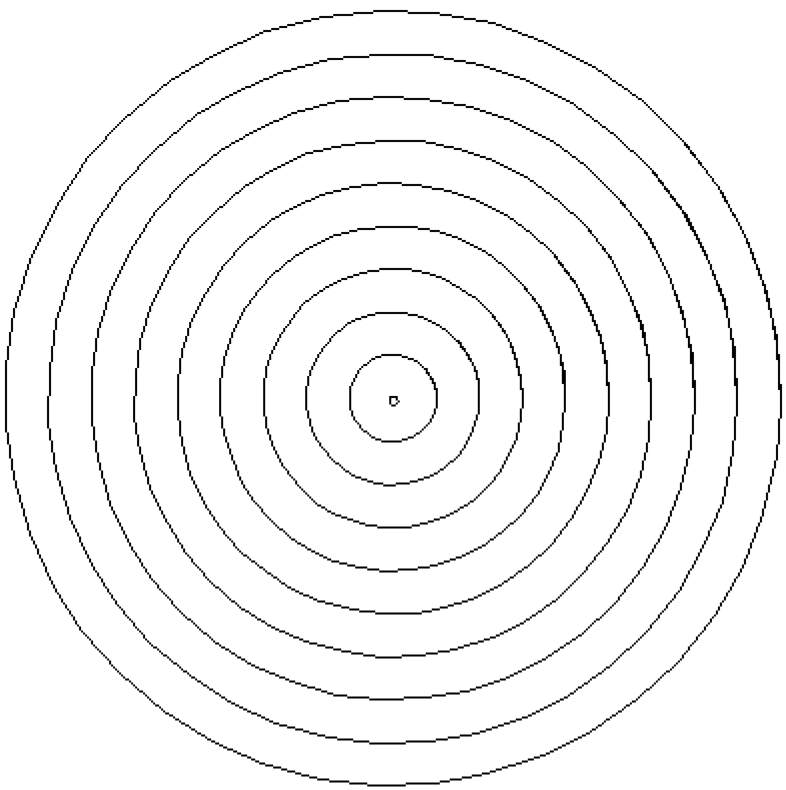
Michael's Sketch

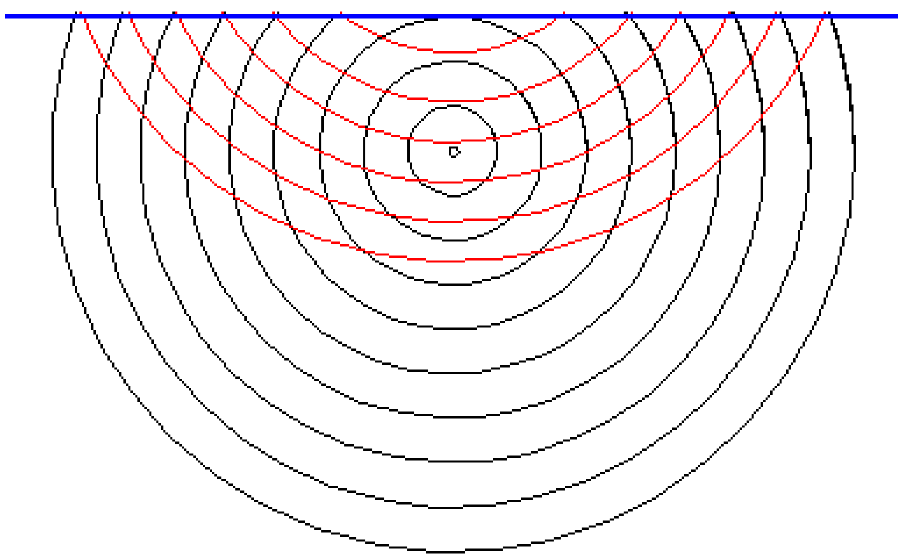


explanatory slide

My husband, Michael Robinson, is also my officemate at UPenn. He drew this sketch on our office chalkboard in order to explain this problem to a collaborator there. The fan shape indicates a wave front which has been bent by reflections due to three barriers, also drawn.

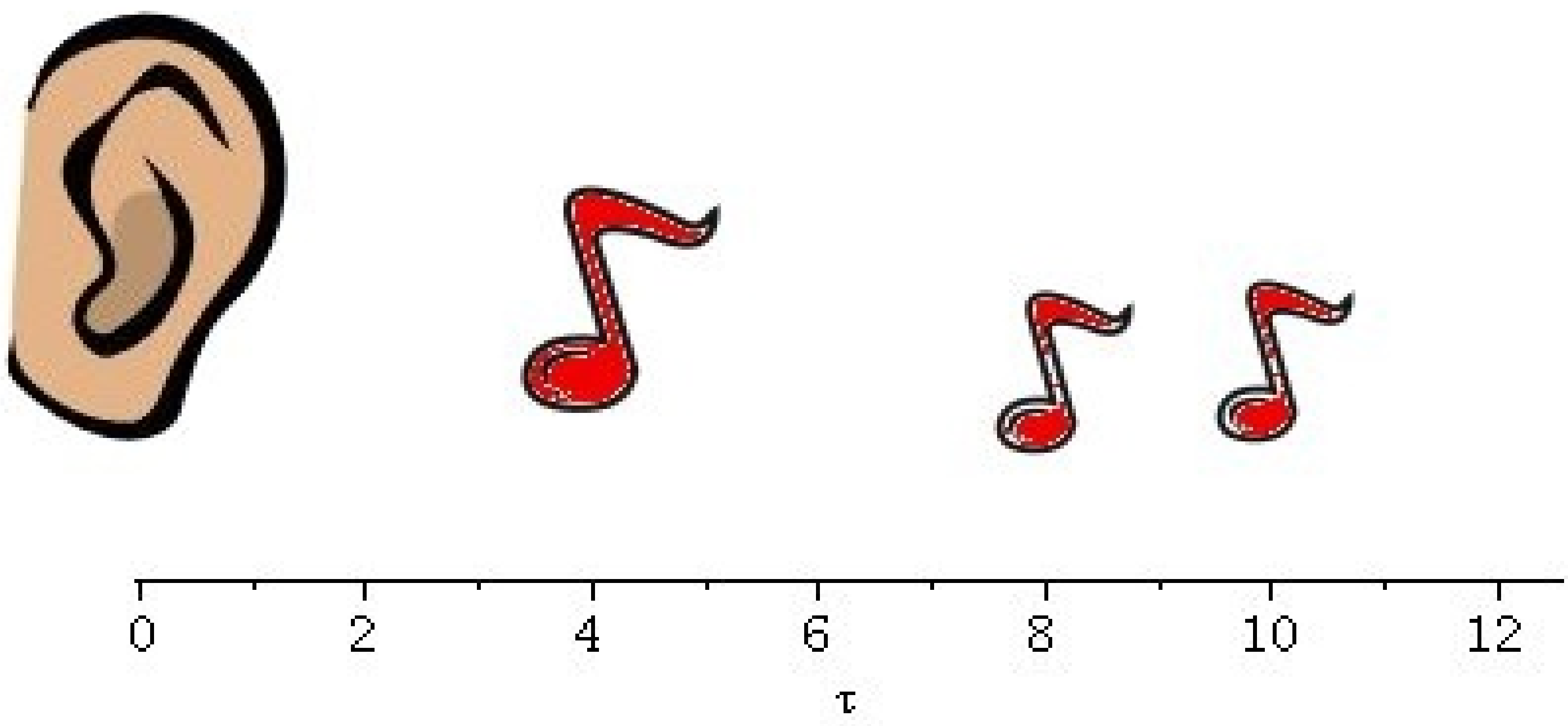
Donna's Sketch





explanatory slide

Here is another similar sketch, with just a single barrier, a wall. You can see how the circular wave front responds to a wall.


0
2
4
6
8
10
12
τ

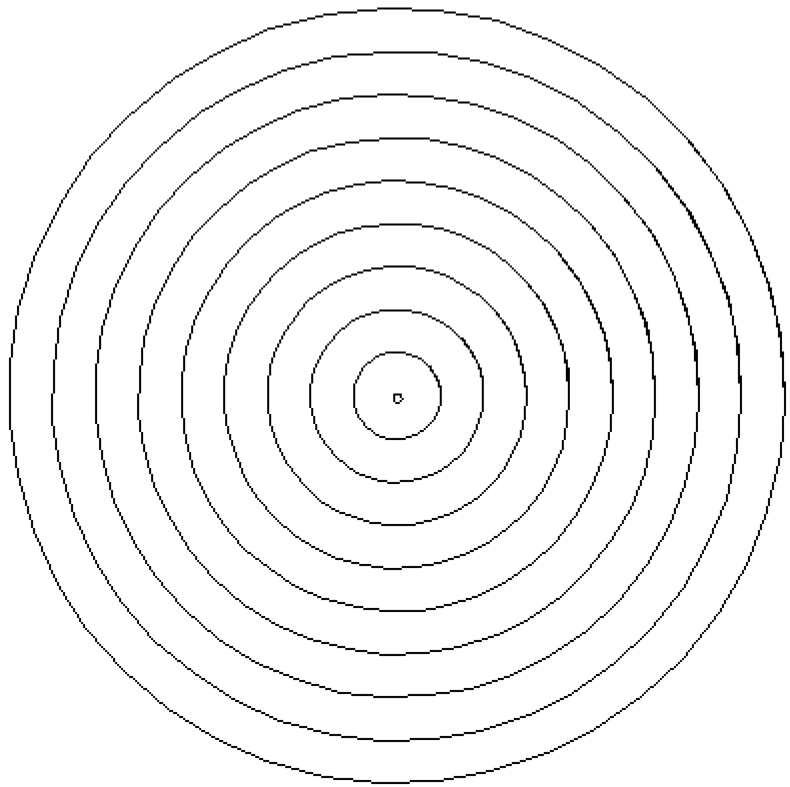
explanatory slide

This slide indicates that at a given position in the plane, you may hear a direct signal at t=4, and reflected signals at t=8 and t=10. This invites (at least) two types of questions.

I. If you consider only the time differential between the first two pulses heard, can this information be used to localize a source? (Assume 2 sources, for example.) What do level curves of time differential look like?

II. In this situation, a pulse is heard three times. Assume the very closest neighbors also hear the pulse three times. What is the neighborhood in which this is true? The analysis of “number of times a pulse is heard” causes the entire domain to be broken into numerous “cells”. What types of cells result from this process?

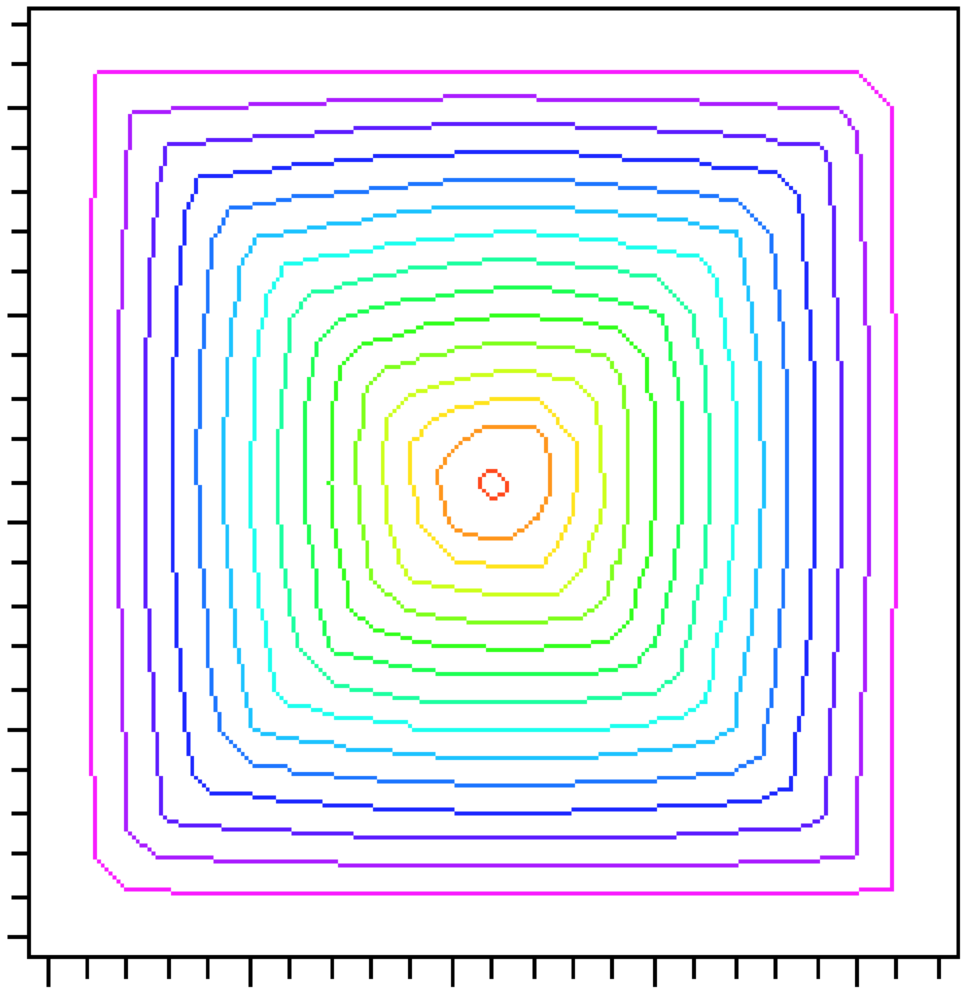
Level Curves of time differential: circle



explanatory slide

In the simple geometry of a circular room and a source at the center of the room, there is no second pulse heard on the wall. Very close to the wall, the reflected pulse is heard immediately (nearly zero differential) while the echo takes the longest to reach the center. This sets up concentric circles. (Again, note that I'm only allowing a single bounce from each ray originating from the source.)

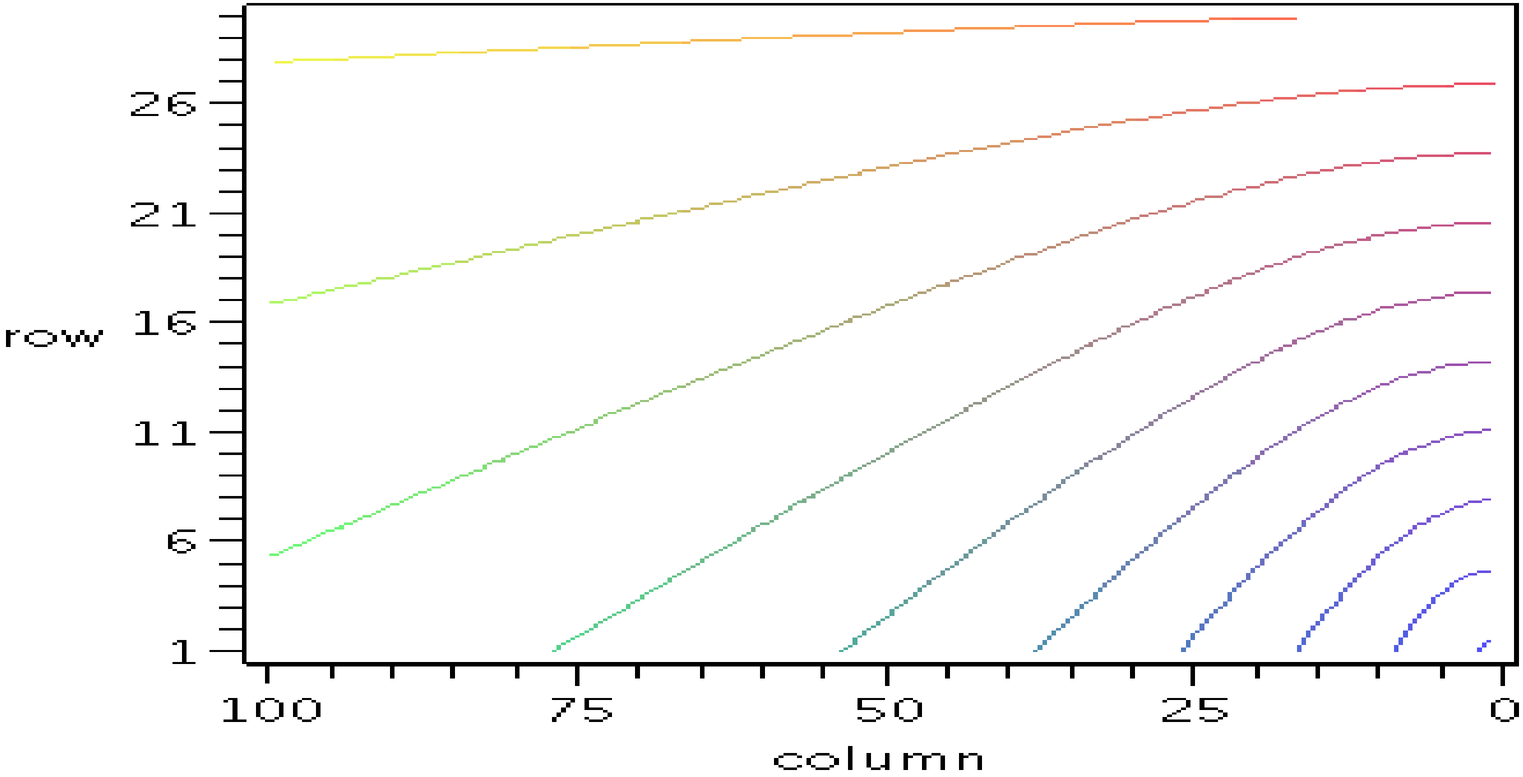
Level Curves of time differential: square



explanatory slide

This is the same thing as the previous slide, except it's actually calculated on Maple, and it is for the square room, with a source nearly at the center. Note the similarity to the circular room. This begs the attention of our topologist friends! I'm investigating the geometry, in part, to fuel the topological intuition.

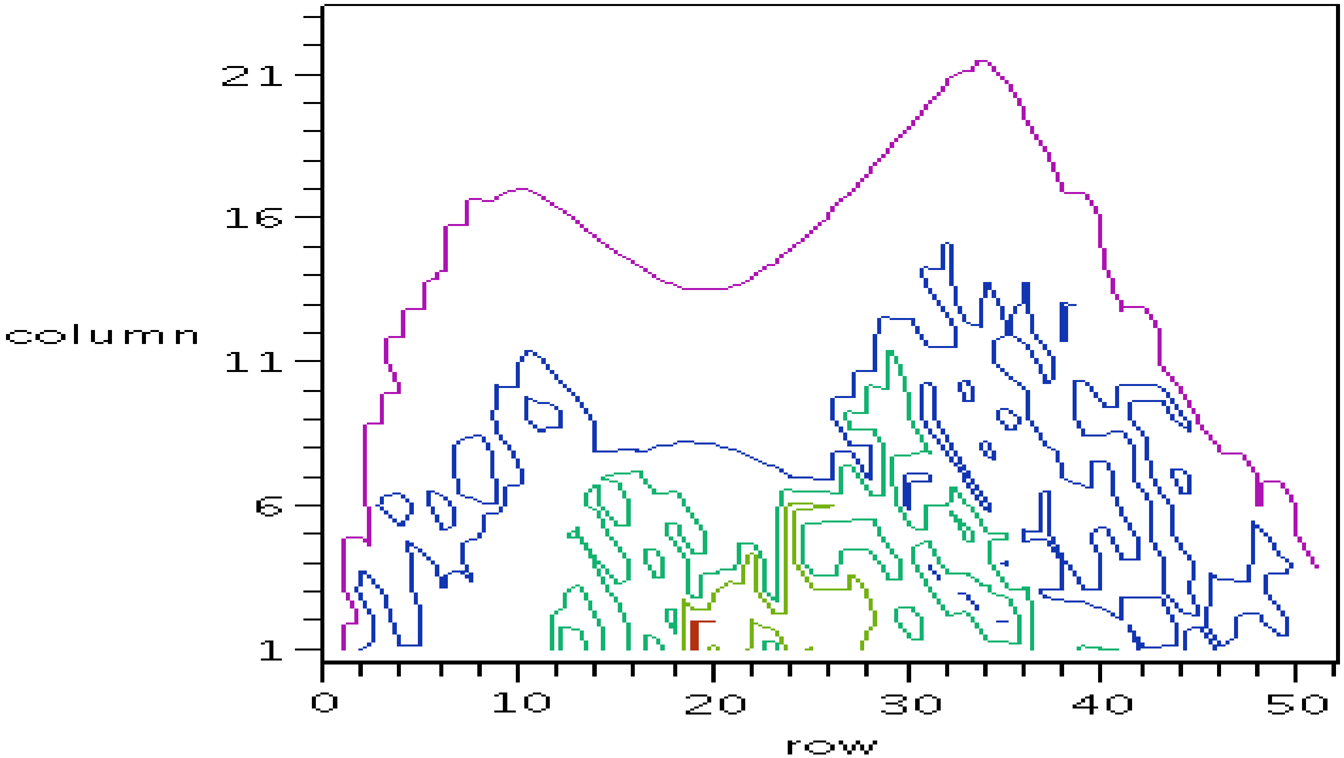
Level Curves of time differential: Half Plane



explanatory slide

In this geometry, we have a lower half-plane with a source in the lower right corner.

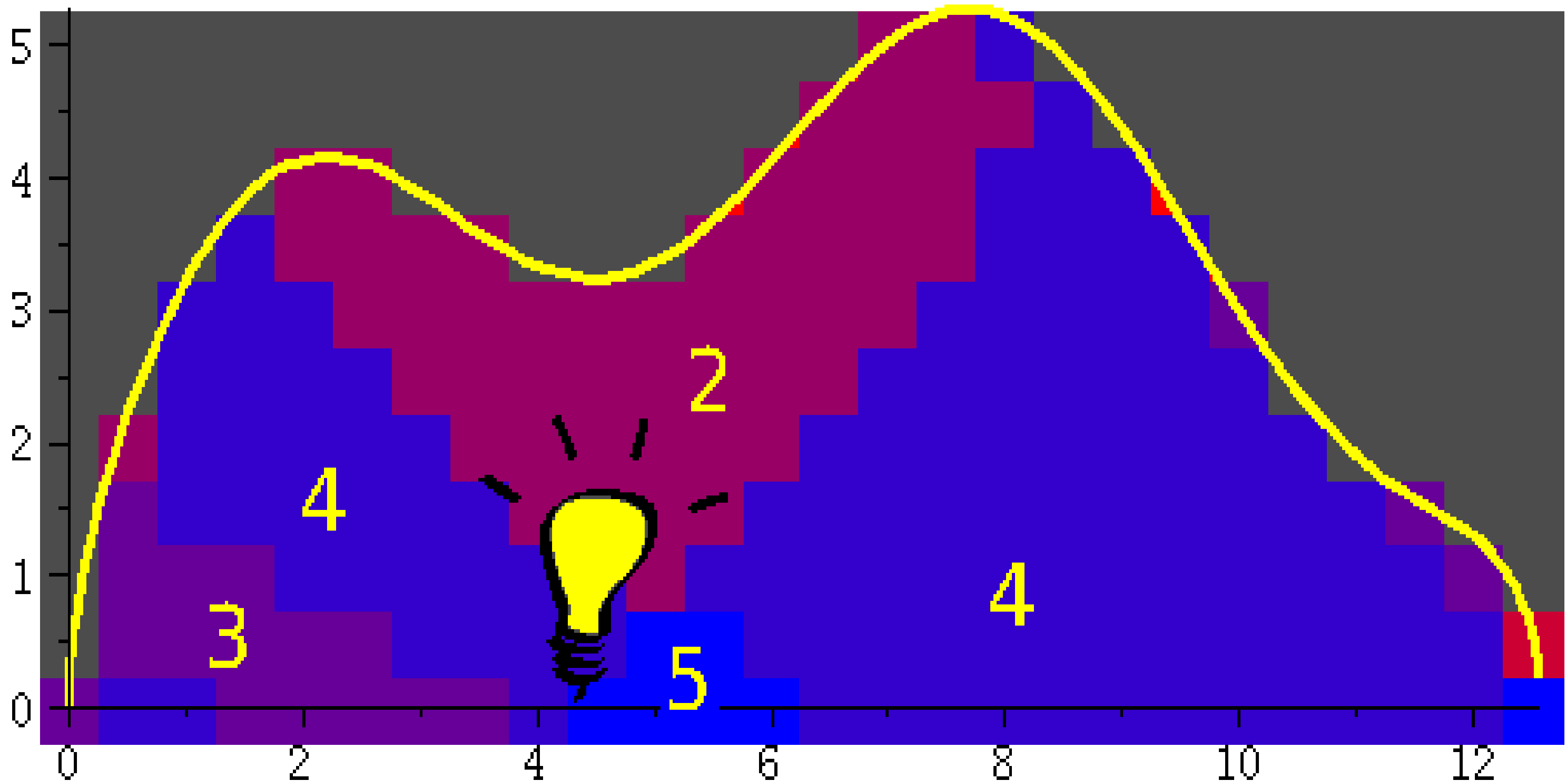
Level Curves of time differential: Bumpy Function



explanatory slide

This slide is included to show that sometimes the numerics are an issue. This plot should contain level curves.

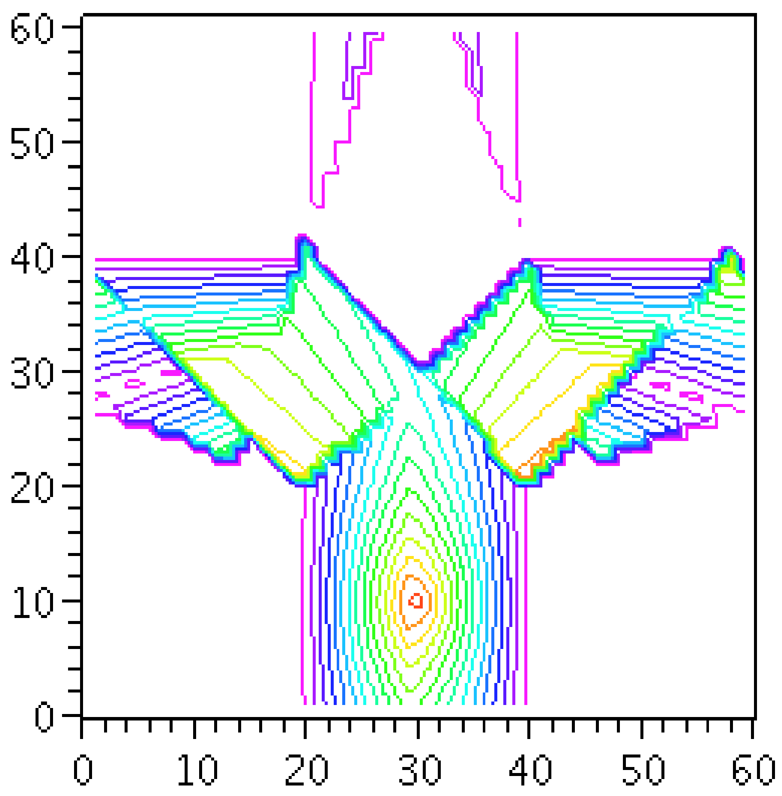
Cellular Decomposition: Bumpy Function

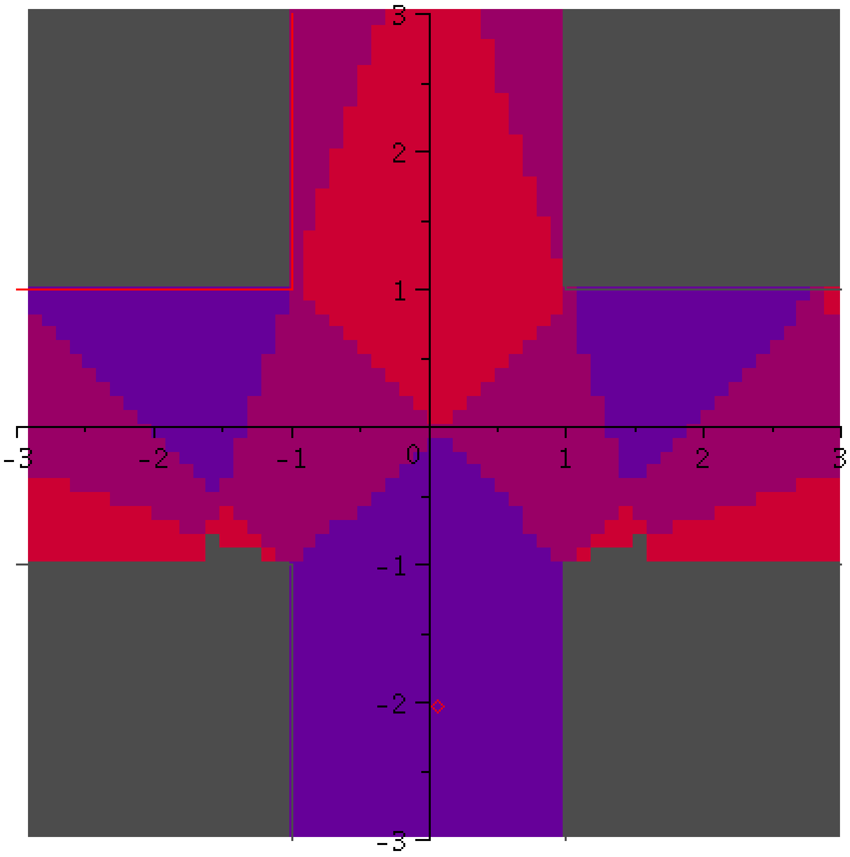


explanatory slide

Here is the cellular decomposition of the same region. One thing to note is the transition between cells is often +2 or -2, rather than +1 or -1 as you might expect. Initially, I ignored this, but I later decided this was important to investigate and explain.

Level Curves & Cellular Decomp: Cross

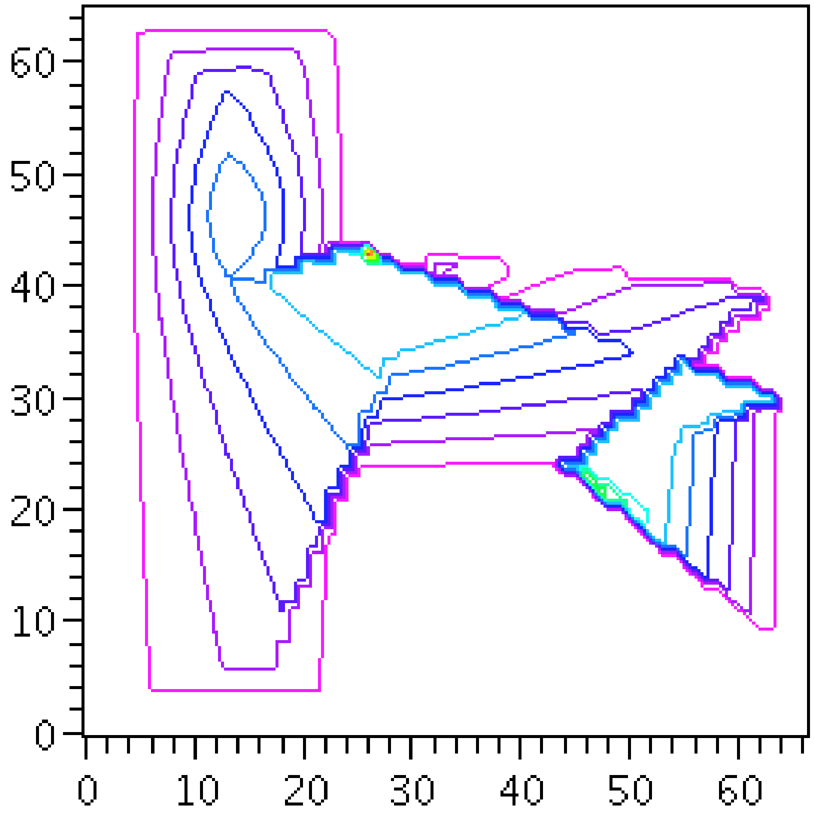


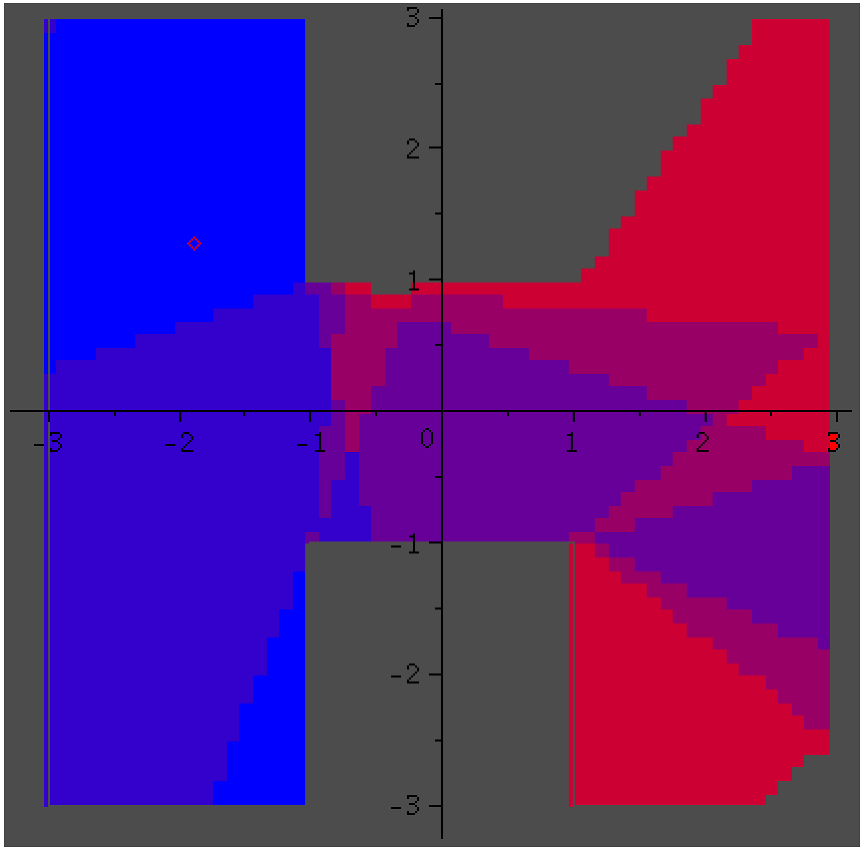


explanatory slide

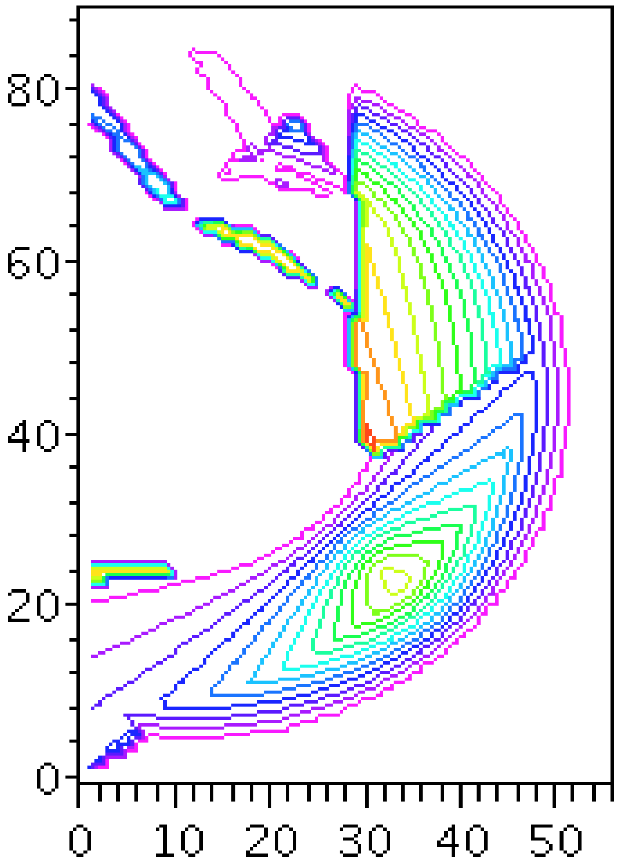
This and following slides give level curves for time differential between the first two pulses, together with cellular decomposition for several geometries. Note that the cells are echoed in the level curves. In other words, each cell can in some way be considered its own geometric region which may or may not transition smoothly to the next cell.

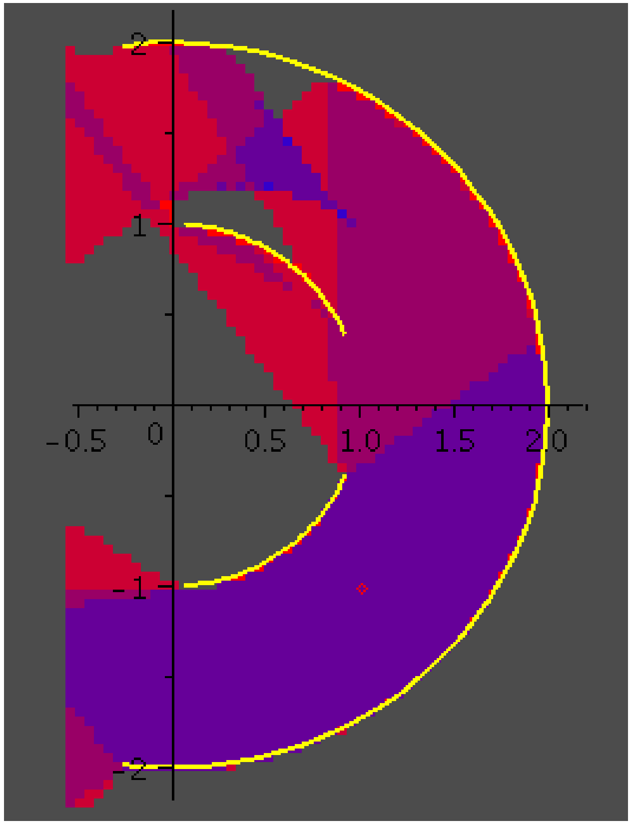
Level Curves & Cellular Decomp: H Region



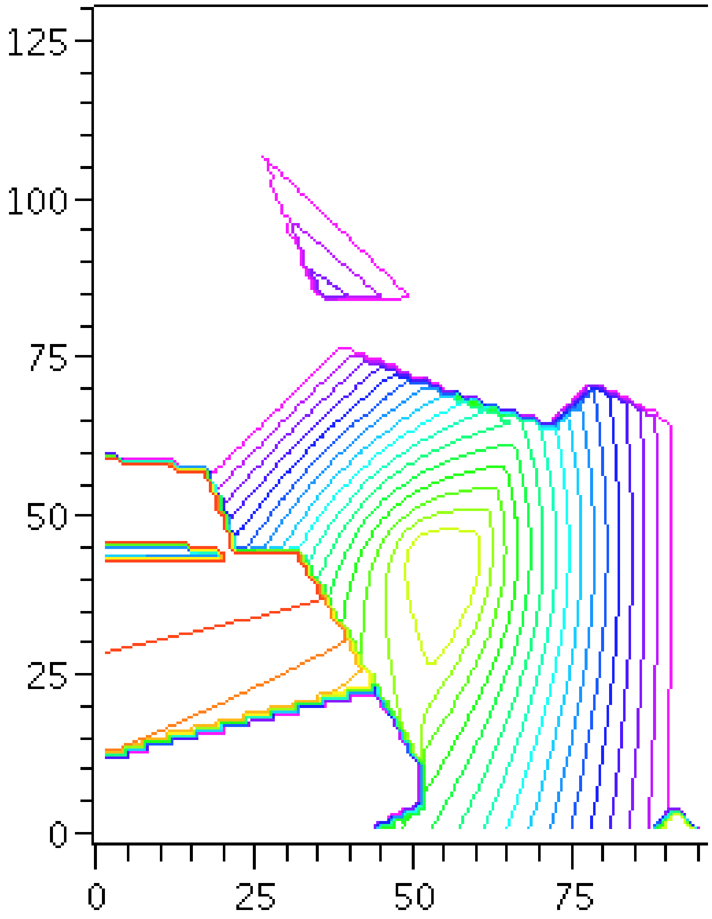


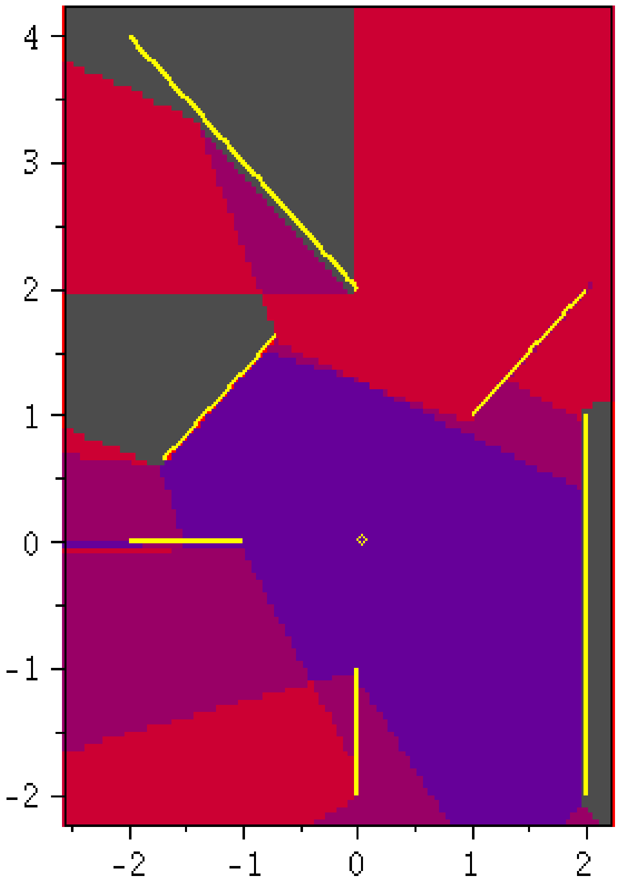
Level Curves & Cellular Decomp: Half-eaten Doughnut



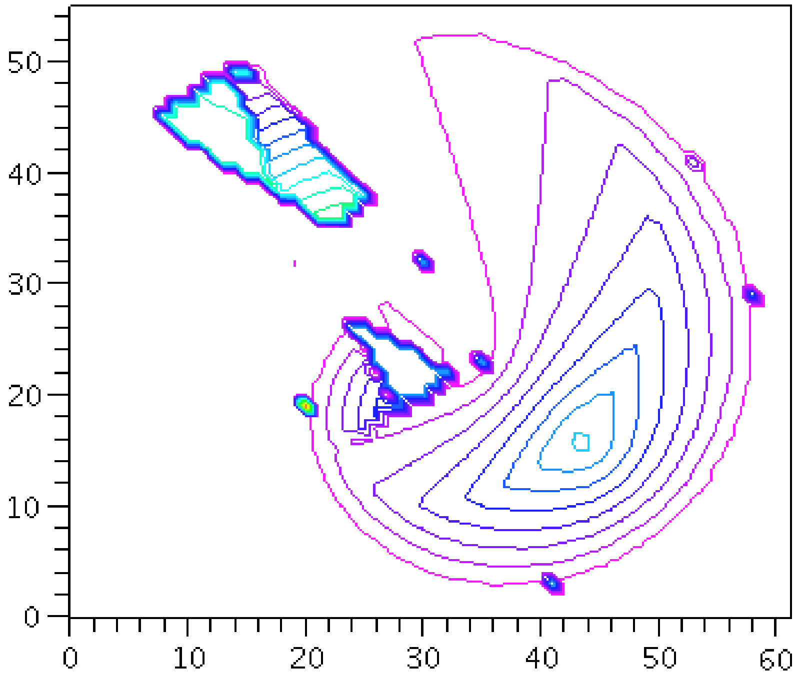


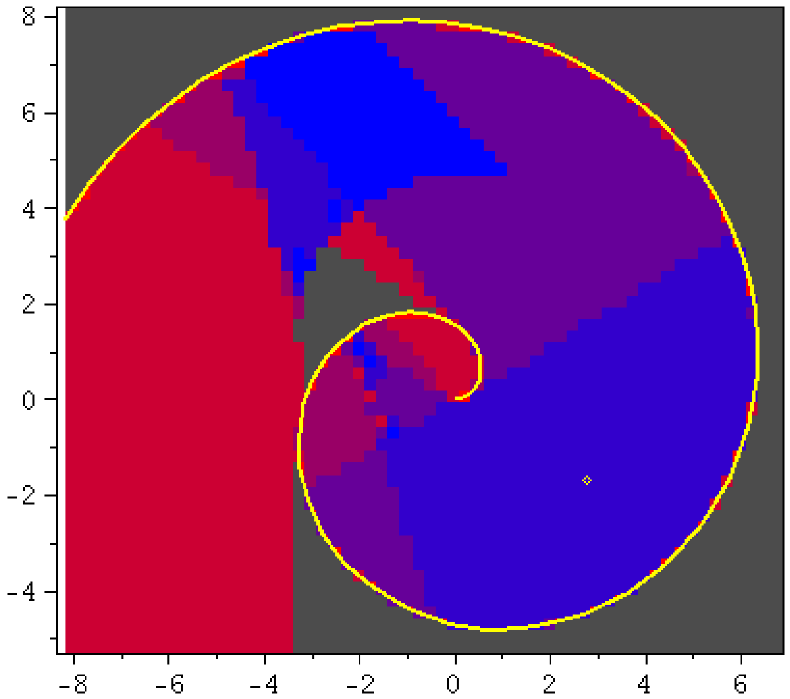
Level Curves & Cellular Decomp: Set of Lines





Level Curves & Cellular Decomp: Spirals.....



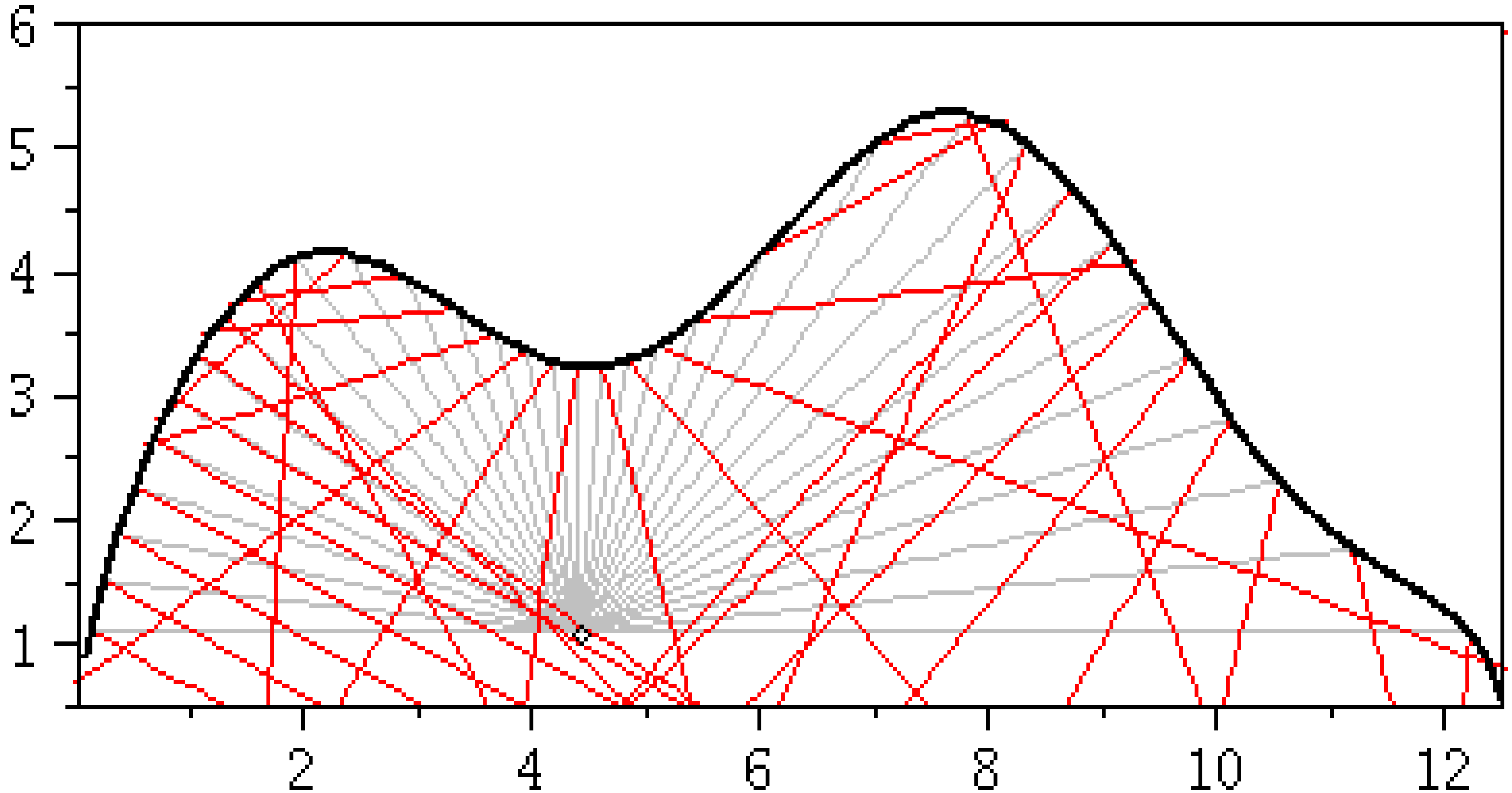


explanatory slide

Those of you who know me realize that I enjoy spirals. (My thesis was on cubic spirals.) So, most of the remaining examples today will be variations on a single spiral geometry. The source will move throughout.

One thing that caught my eye was this “strange bright blue wedge shape” in the upper middle portion of the cellular diagram.

STRING ART!

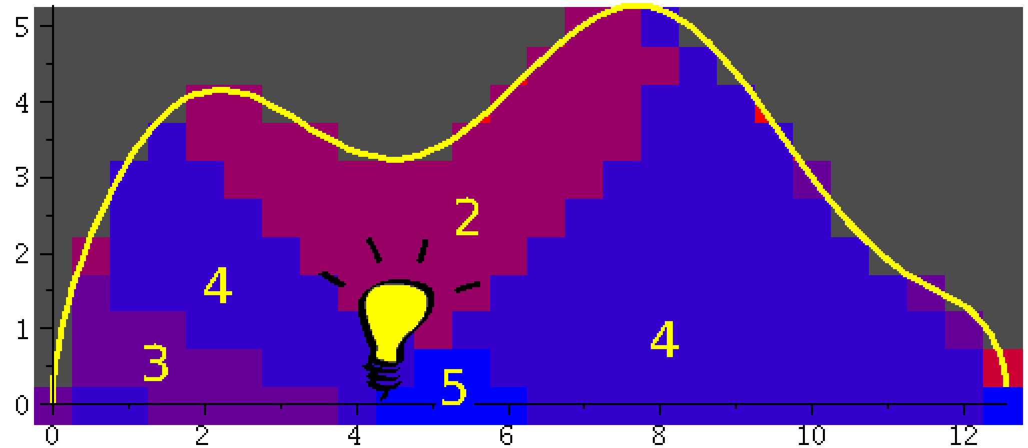


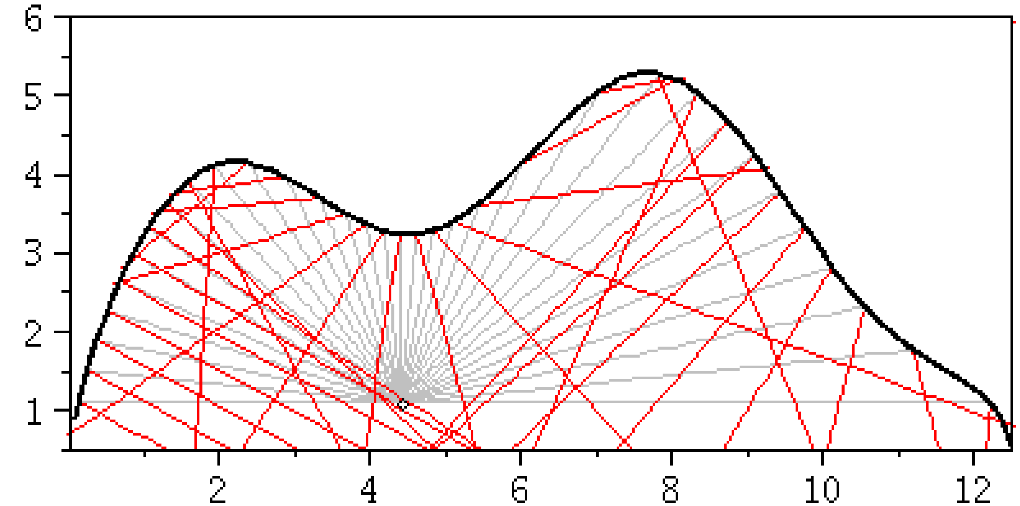
explanatory slide

To figure out why this was happening, I added another step in my analysis. I traced out the rays to see where they were going, and tried to figure out why various phenomena were occurring.

The diagrams which resulted looked like “string-art”.

Compare Cell Decomp & String Art: Bumpy Function

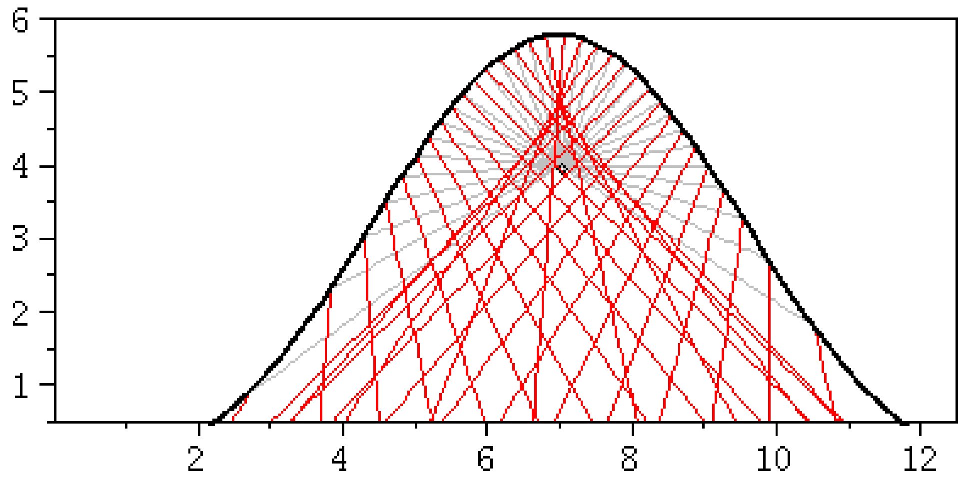


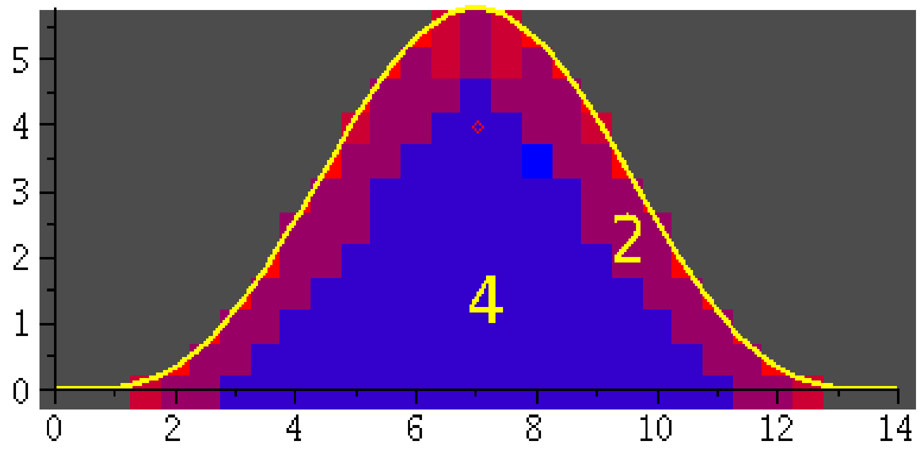


explanatory slide

Comparison of these two diagrams begin to provide a reason for the +2/-2 phenomenon. Note that an envelope of curves (in the string-art diagram) is actually forming the boundary between cells here.

Cellular Decomp & String Art: Bell Curve

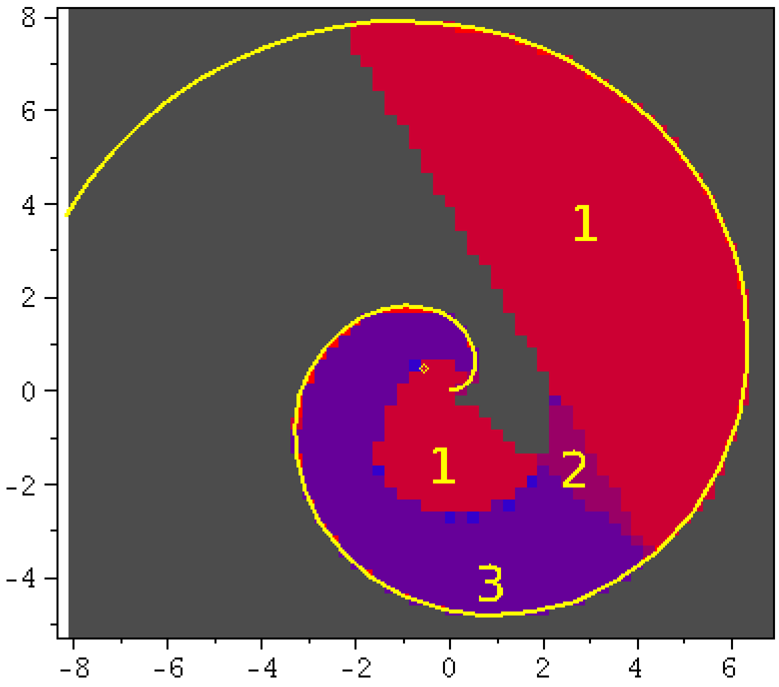


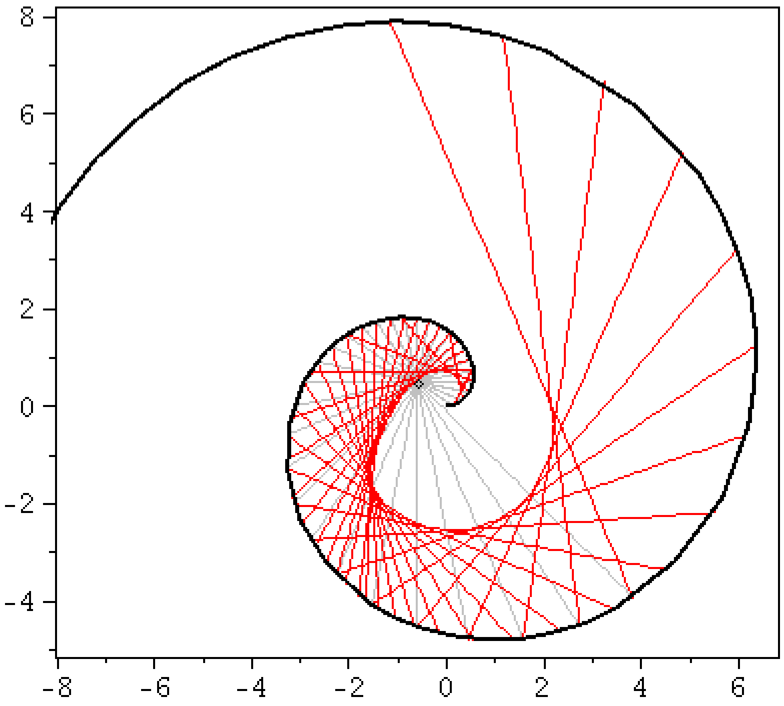


explanatory slide

Also in this case, there is an envelope in the string-art diagram causing a +2/-2 boundary in the cellular decomposition diagram. The envelope is a codimension 1 caustic, while the cusp at the top is a codimension 2 caustic.

Spiral Series: 1 of 11

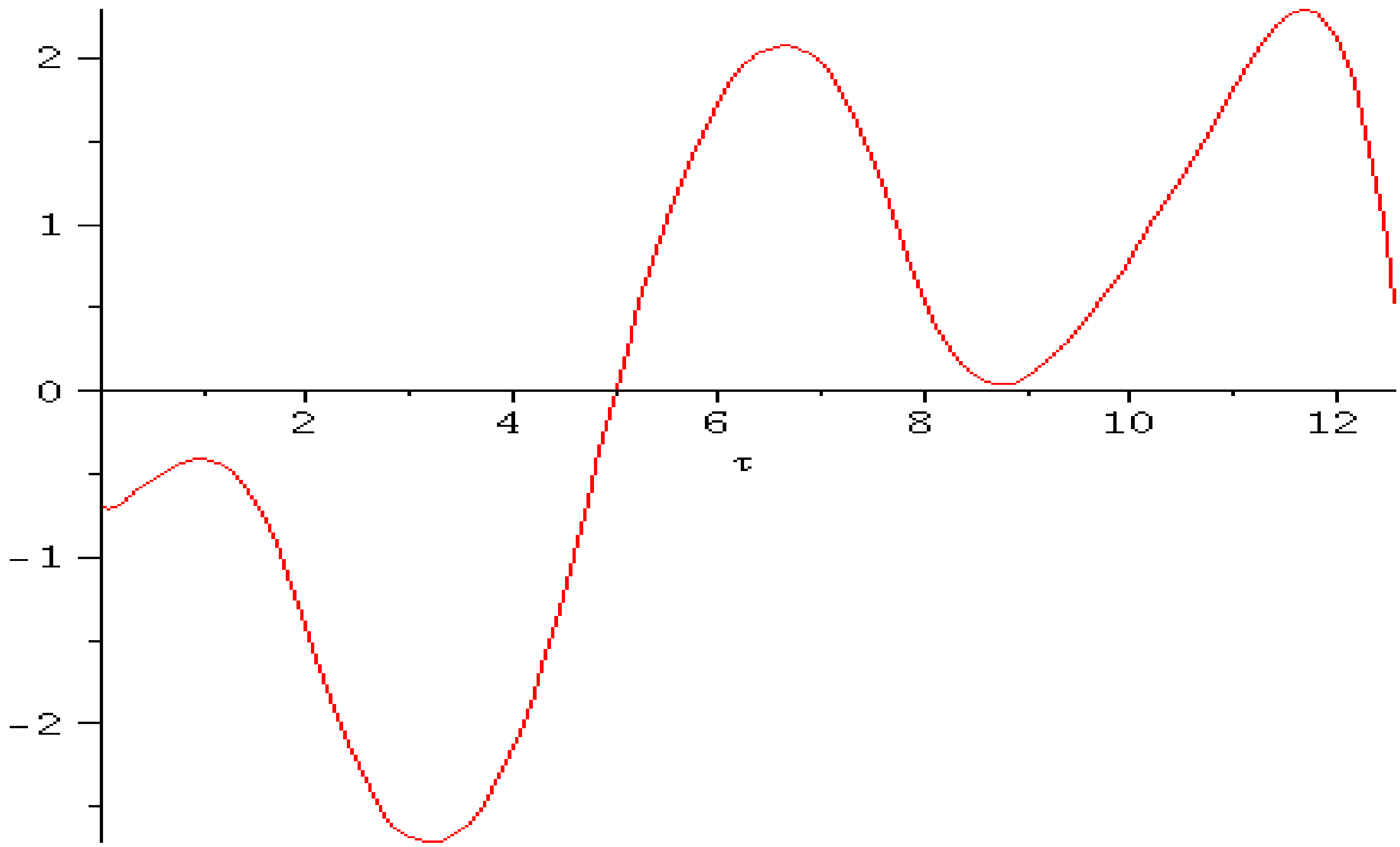




explanatory slide

This is the first of several slides which will pass quickly. The source is moving. Note the relationships between the two diagrams.

Quick Look: The degree two root on the envelope

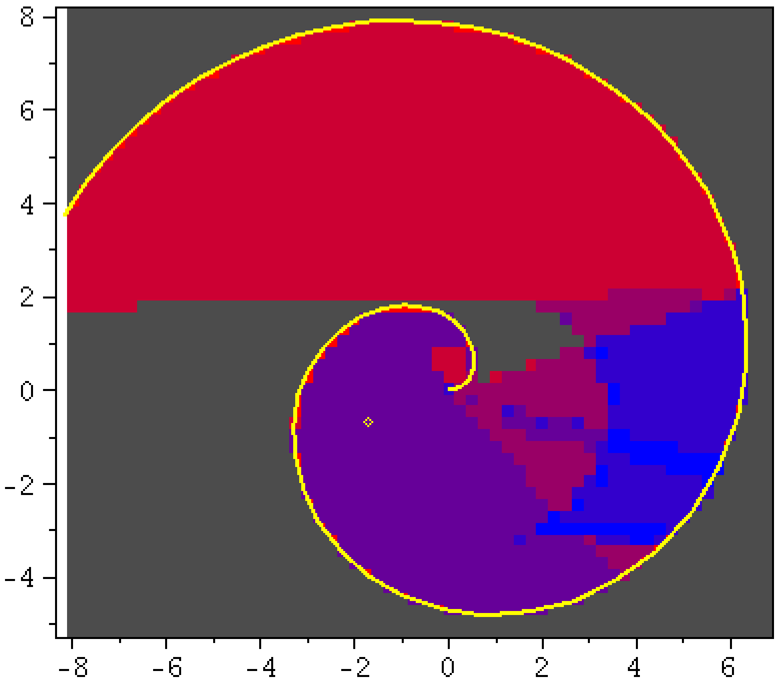


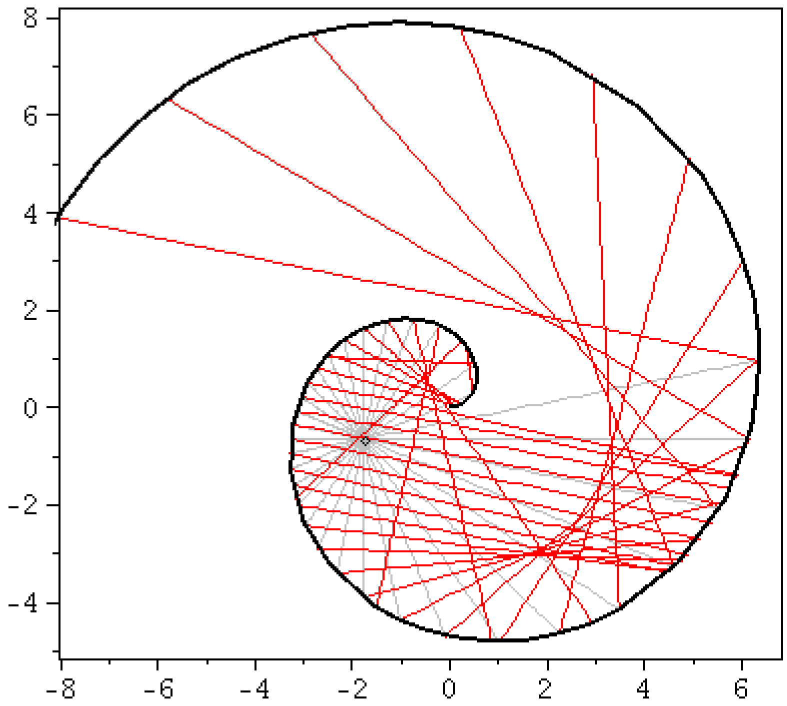
explanatory slide

This is a quick slide to show you the numerics behind this. The order two root is the root where the +2/-2 cellular crossing occurs, and it's on that envelope in the string-art diagram.

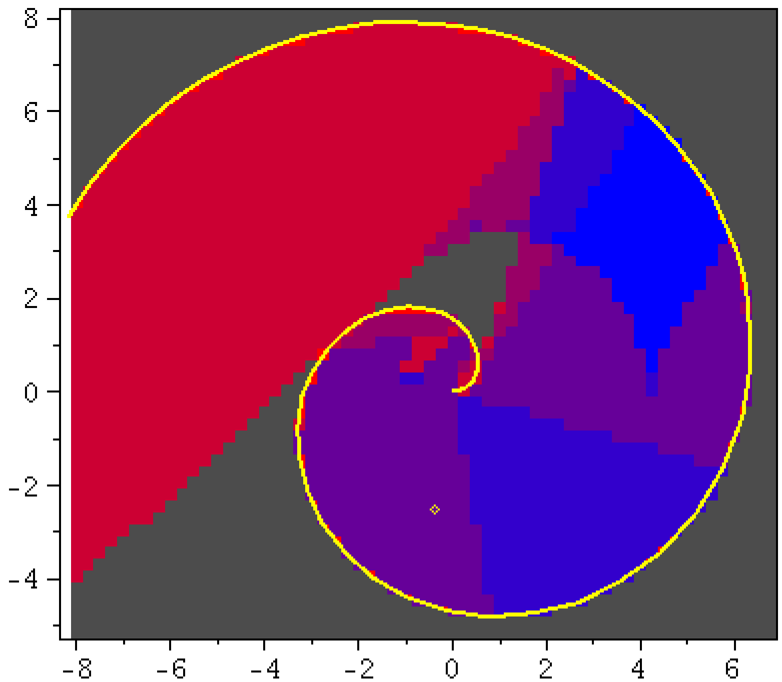
This function's independent variable is the parameter used to draw the spiral, and the dependent variable tells how close we are to getting a reflection. This function assumes a fixed source and a fixed “listening” point. Each root indicates a single-bounce reflection heard at the “listening” point.

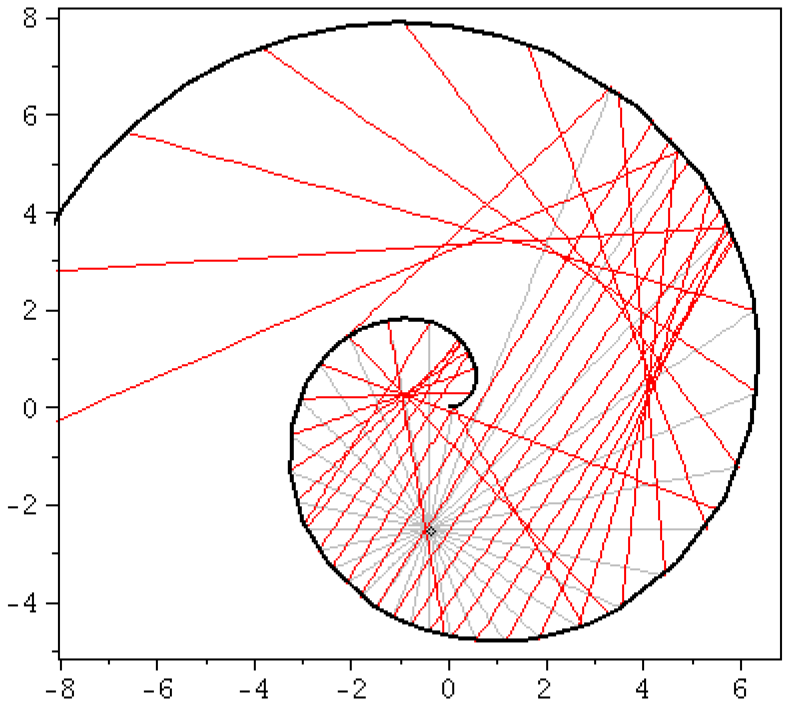
Spiral Series: 2 of 11



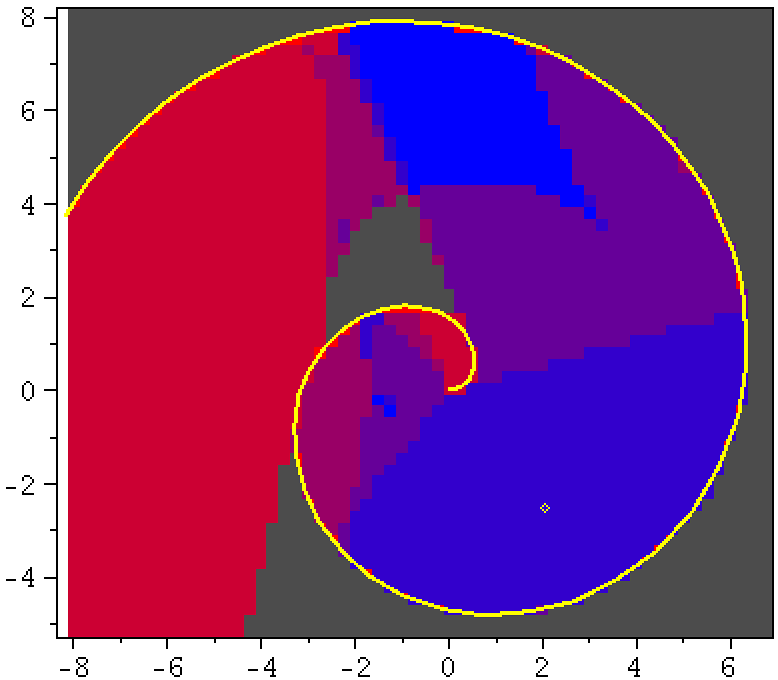


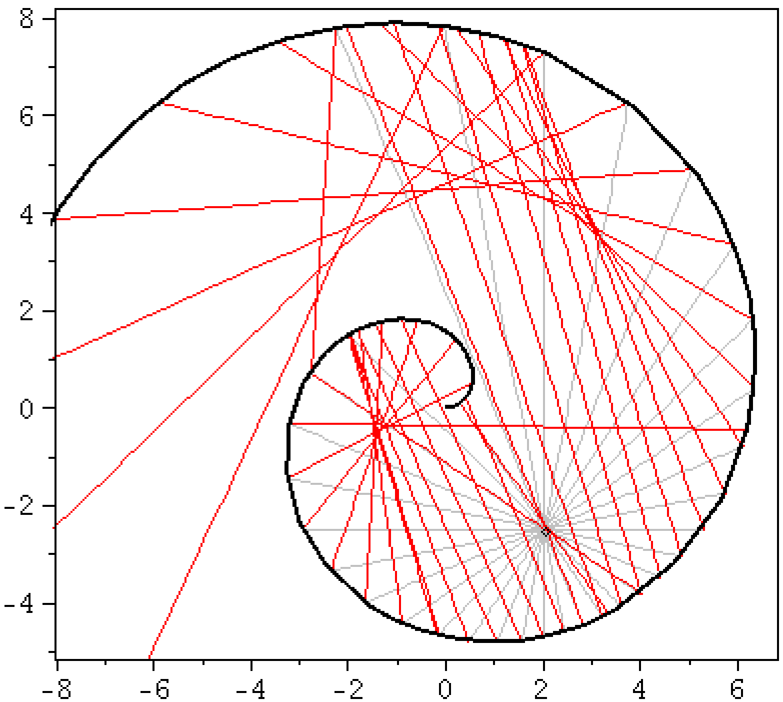
Spiral Series: 3 of 11





Spiral Series: 4 of 11

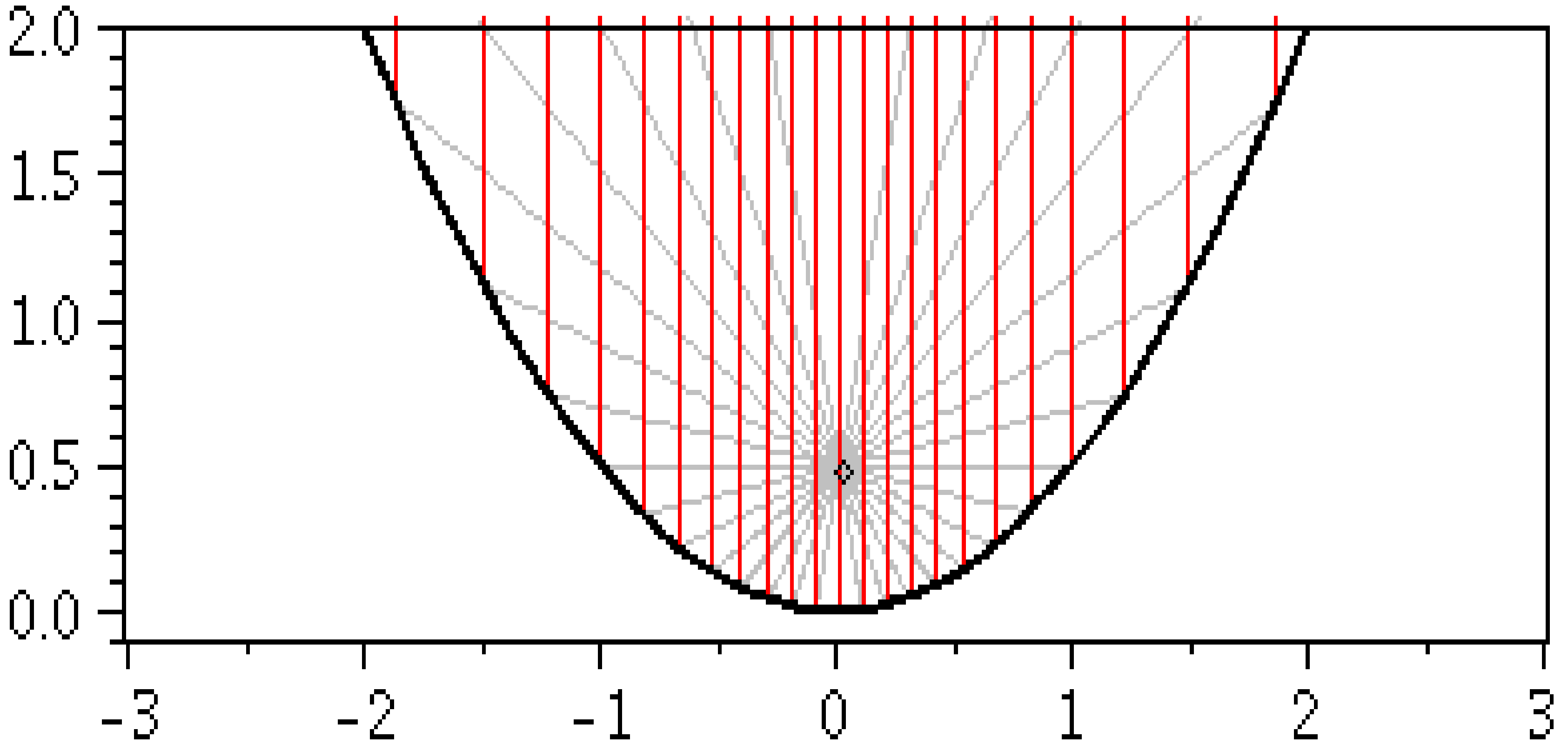




explanatory slide

You can see here the “blue wedge” coming into play again. Now, with the “string-art” diagrams, you can see what's actually causing this. Part of the answer is that the envelope is creating this “spiral shadow”, but the other thing causing the wedge to show up is this “channel effect” coming off the opposite side of the spiral. Notice there are nearly parallel lines coming off of one side of the curve, causing a “channel” of sound/light to be created.

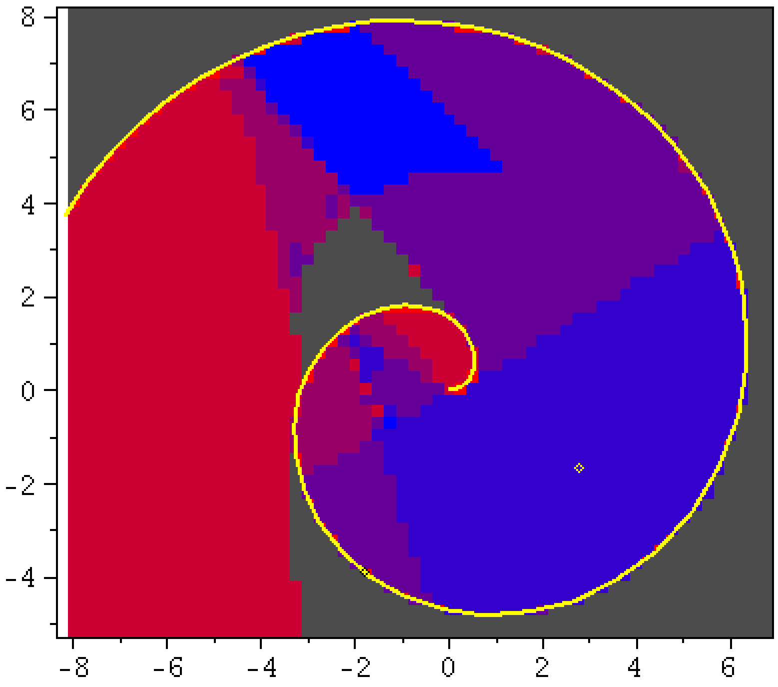
The Parabola Effect

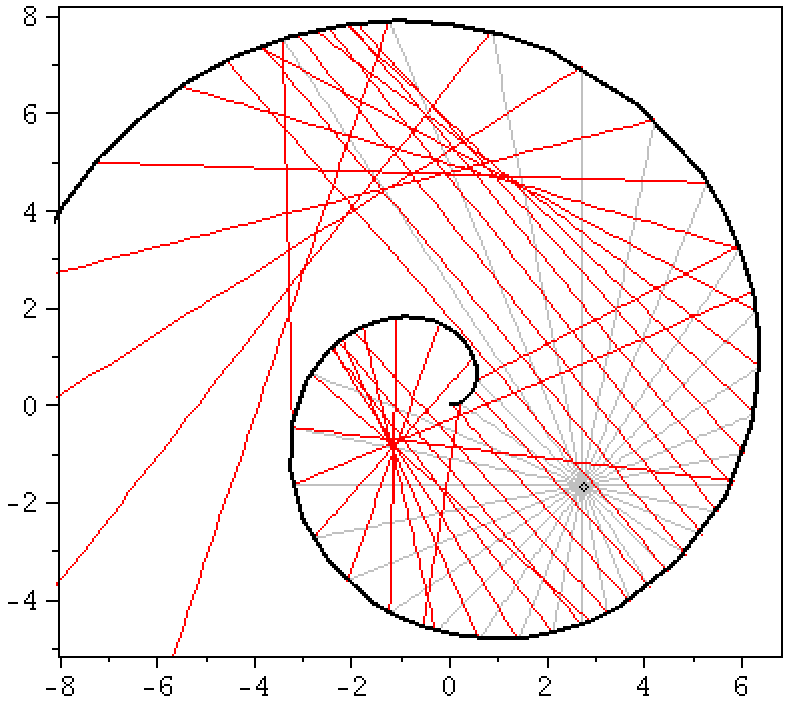


explanatory slide

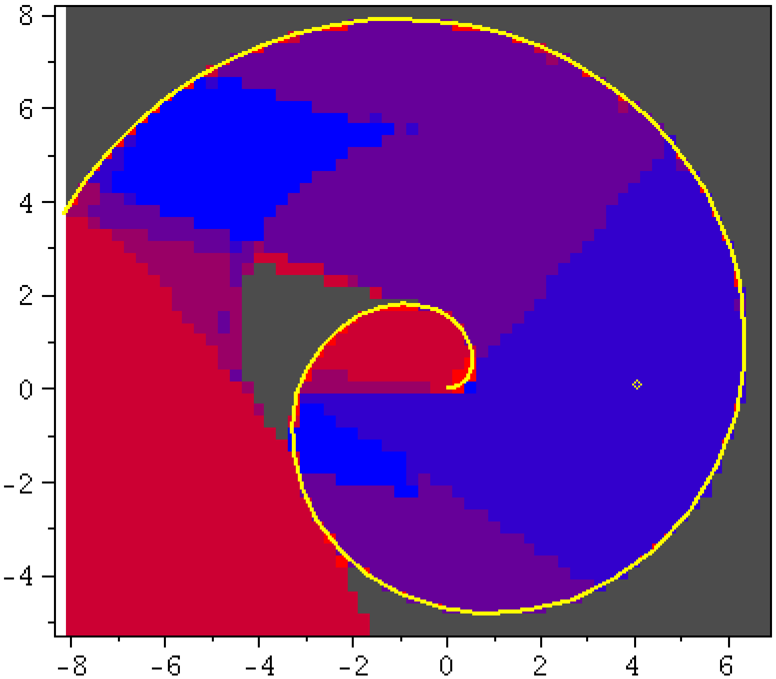
This “channel effect” comes from the nearly parabolic effect of the spiral with respect to that source. Wherever the spiral approximates a parabola well, relative to the source, a “channel” will form.

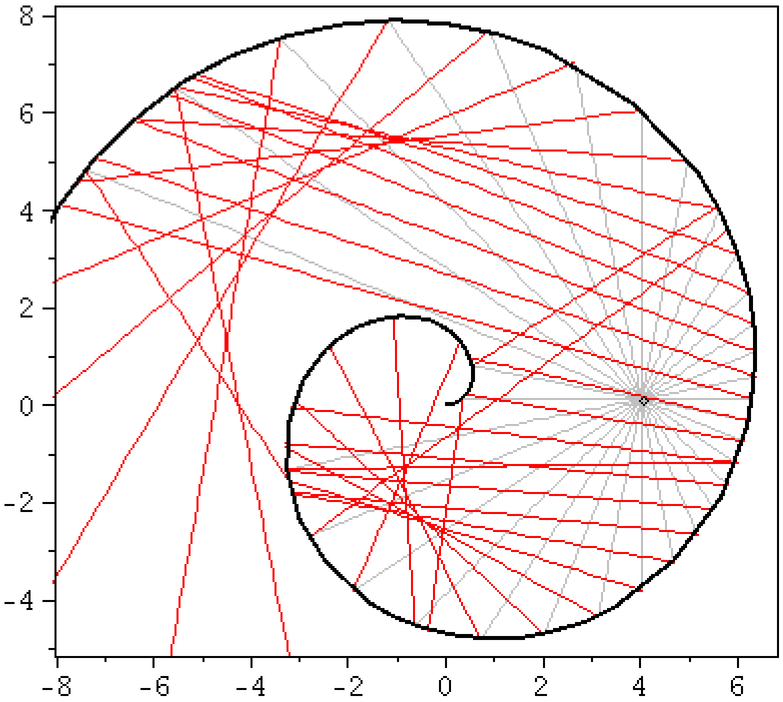
Spiral Series: 5 of 11



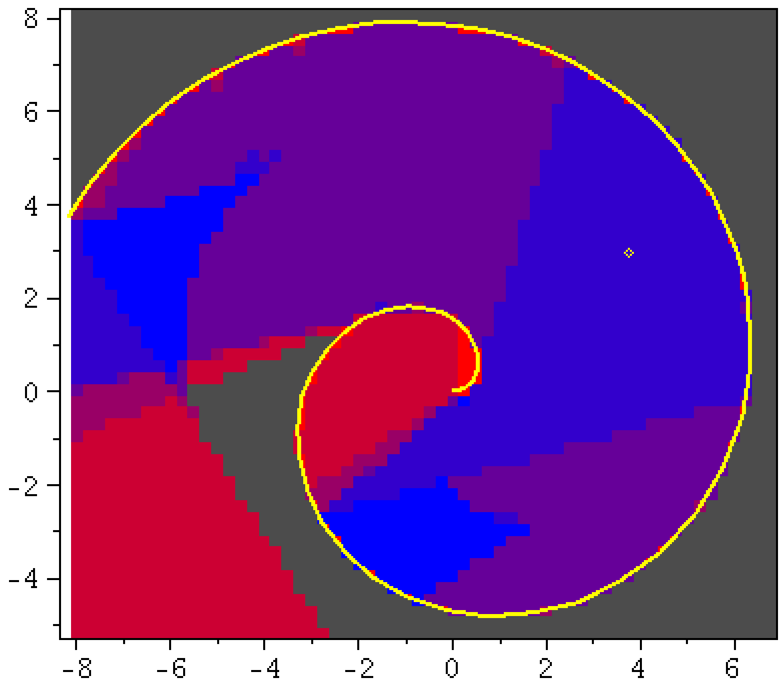


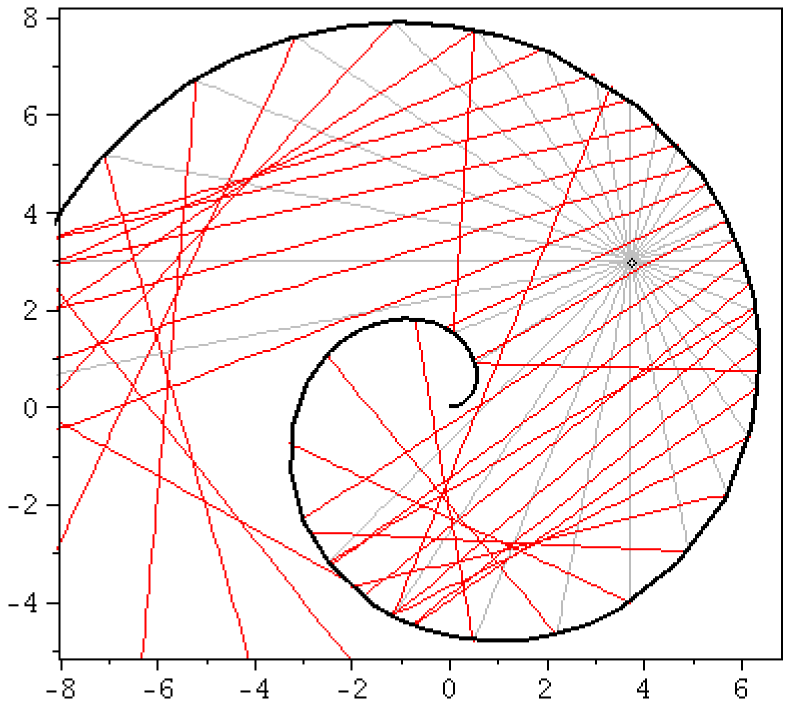
Spiral Series: 6 of 11



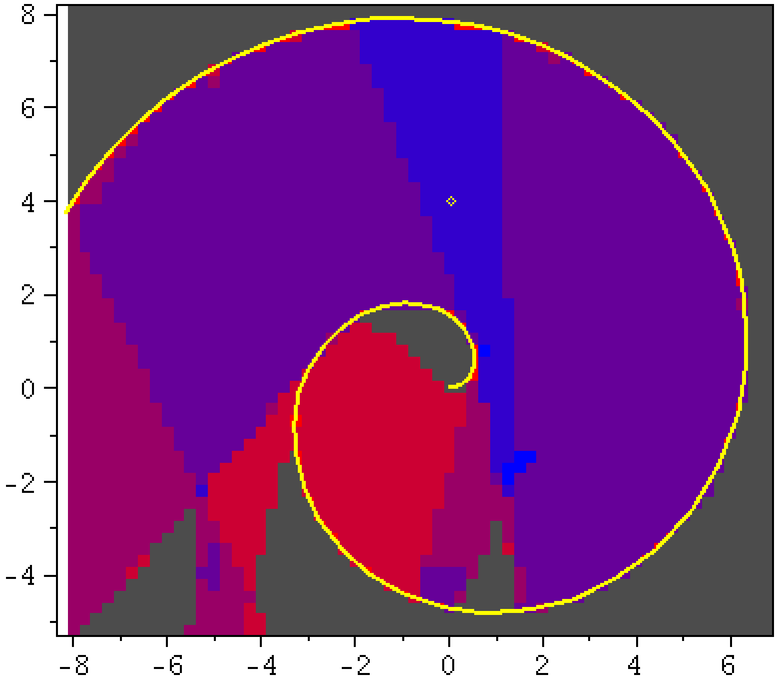


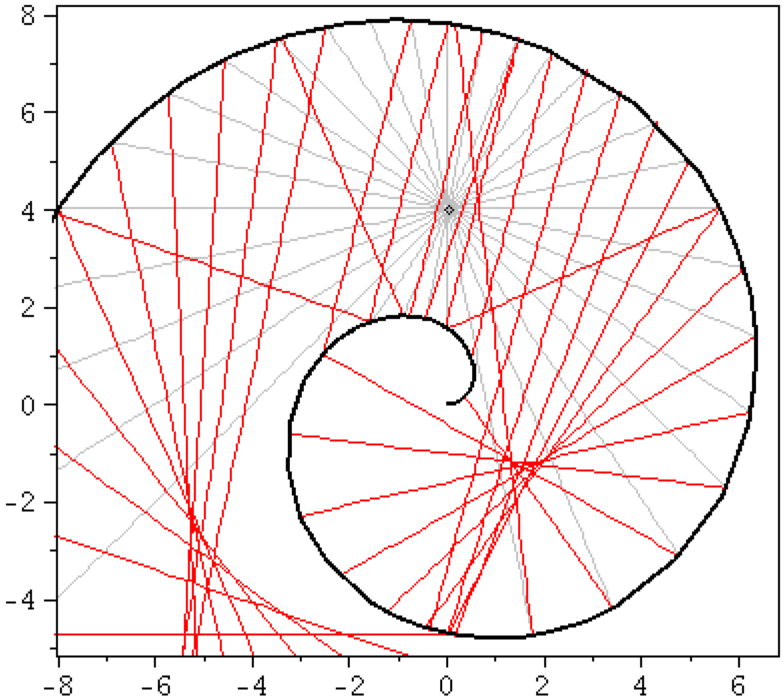
Spiral Series: 7 of 11



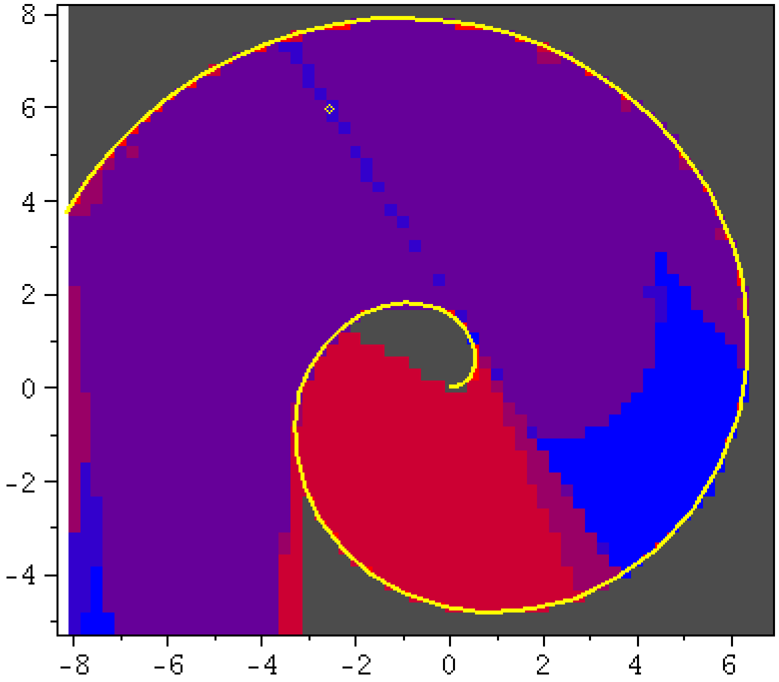


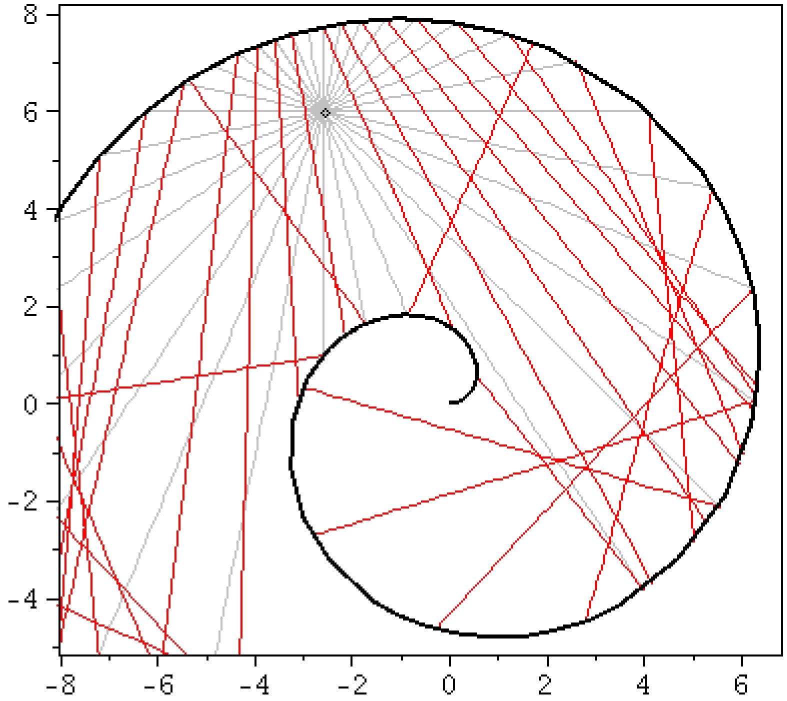
Spiral Series: 8 of 11



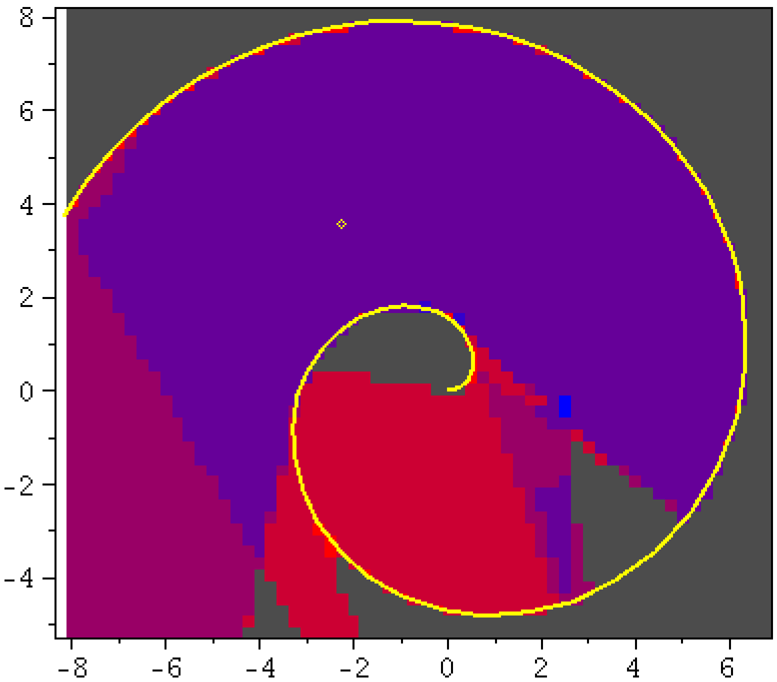


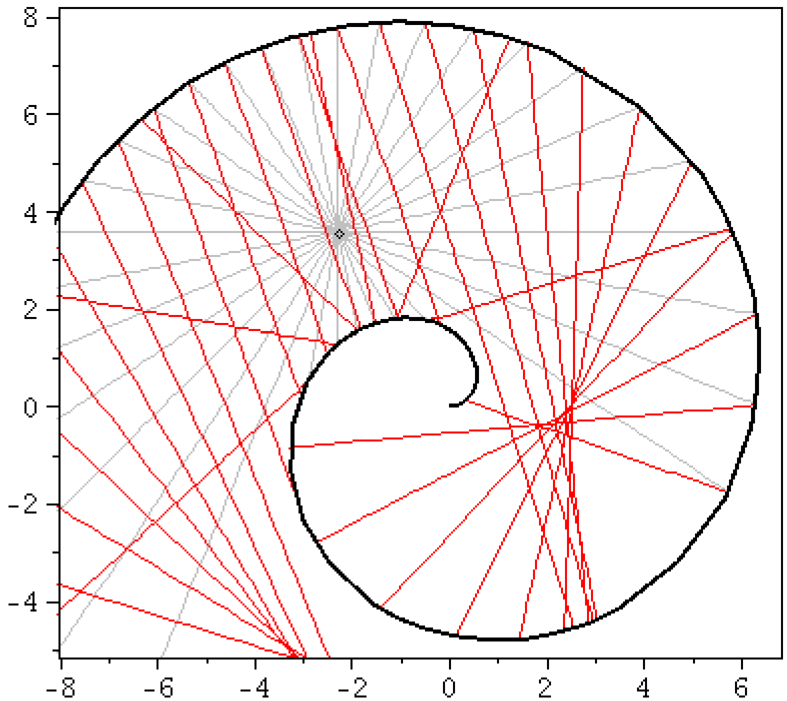
Spiral Series: 9 of 11



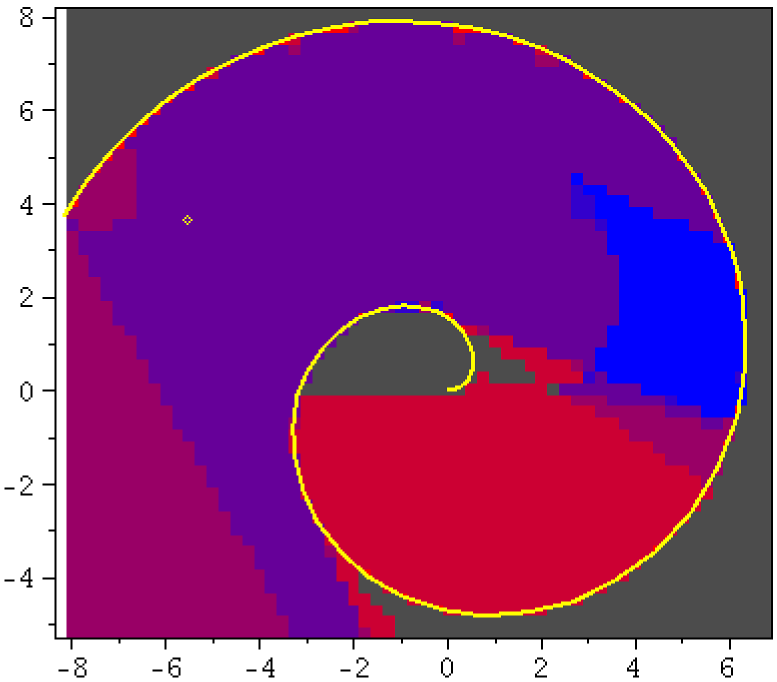


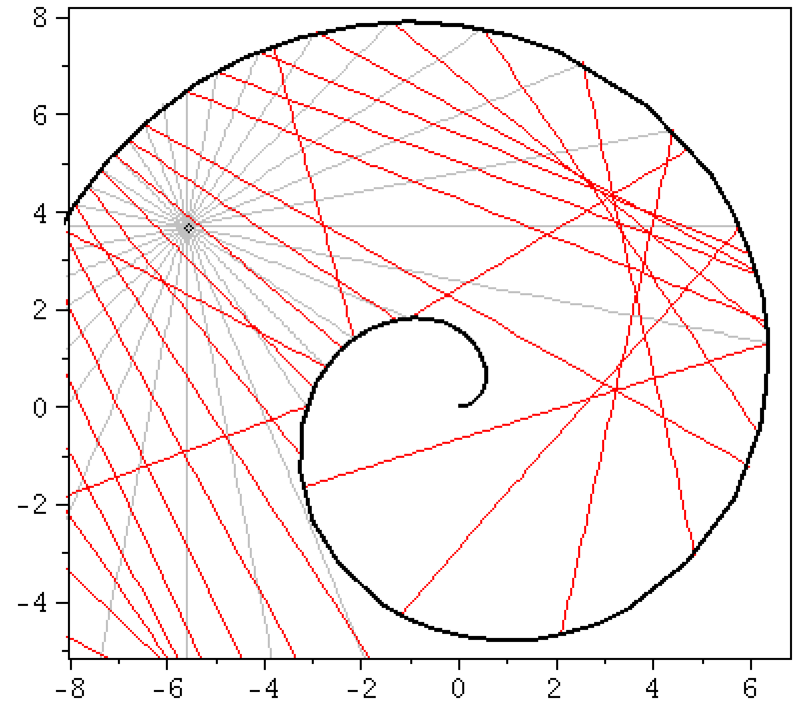
Spiral Series: 10 of 11





Spiral Series: 11 of 11





Possible Future Work:

- Localization based on 2+ sources
- Topological generalizations
- Move code away from MAPLE and over to C
  - faster, so less ATARI-like images
- 3D
- More ray-launching with adaptive algorithms
- Investigate numerics of this problem
  - lots of fiddling with constants: make rigorous

explanatory slide

I have only been working on this for about 2 months, so the list of future work is much longer than what I've accomplished. Of course, as this progresses, that ratio probably won't change.

Feel free to contact me at prof.dietz@gmail.com if you have comments!