Economics 301
Homework #7
October 26, 2001

1. Duane breeds parrots for a living. He has discovered that the production function for parrot chicks, Q, is

$$Q=1/2\ K^{1/2}L^{1/2}$$

Where K is the amount of capital and L is the amount of parrot food. The price of K is \$8 and the price of L is \$2.

a. What type of production function is this?

b. What type of returns to scale is exhibited?

c. What is the average product of capital?

d. Does capital obey the "law of diminishing returns?"

e. Suppose Duane wants 144 parrot chicks, how much capital and food should be used to minimize costs and what is the cost of producing that amount of chicks?

f. Suppose that Duane is faced with a fixed amount of capital, K=16, and he wants to still produce 144 parrot chicks. How much food should be used to minimize costs and what is the total cost?

2. The Longheel Press produces memo pads in its local shop. The company can rent its equipment and hire workers at competitive rates. Equipment needed for this operation can be rented at \$52 per hour, and labor can be hired at \$12 per worker hour. The company has allocated \$150,000 for the initial run of memo pads. The production function using available technology can be expressed as:

$$Q = 0.25K^{1/4}L^{3/4},$$

where Q is memo pads (boxes per hour), K is capital input (units per hour), and L is labor input (units of worker time per hour).

a. Construct the isocost equation.

b. Determine the appropriate mix to get the greatest output for an outlay of \$150,000 for a production run of memo pads. Also, compute output.

c. Explain what would happen in the short-run (keeping capital fixed) to the appropriate input mix if production were changed to 1,500 units per hour.

Would the input combination be different in the long-run? If so, how would it change? Explain.

3. You manage a plant that mass produces engines by teams of workers using assembly plants. The technology is summarized by the production function $Q = 4KL$, where Q is the number of engines per week, K is the number of assembly machines, and L the number of labor teams. Each assembly machine rents for $r=\$12,000$ per week, and each team costs $w=\$3,000$ per week. Engine costs are given by the cost of labor teams and machines, plus 2,000 per engine for raw materials.

    a. Your plant has a fixed installation of 10 assembly machines as part of its design. What is the cost function for your plant—what will it cost to produce Q engines? What are average and marginal costs of producing Q engines?

    b. How many teams are required for producing 80 engines? What is the average cost per engine?

    c. Now assume the company can vary its use of assembly machines. What is the long run cost function for the plant?

4. Imagine that a firm has the following production function: $Q=4K^2+2L^{1.2}$. Further suppose that the price of labor is w, the price of capital is r, and the price of output is \$8.

    a. Derive the profit maximizing levels of K and L as functions of w and r.

    b. Now suppose that the wage rate is \$8 and the price of capital is \$32. How much of each factor will the firm use?

    c. Is the firm making a profit or loss?

5. Consider a firm with the following production function

$$Y=K^{1/2}L^{1/4}$$

where K and L denote capital and labor, respectively. Suppose the price of the output is \$4. Let w denote the wage rate for labor and r the price of capital.

    a. What are the returns to scale associated with this production function?

    b. What is the marginal product of labor? What is the marginal product of capital?

c. Derive an expression for the amount of capital and labor used by this firm to maximize profits (as a function of w and r).

d. Suppose the wage rate is now $2 and the price of capital is $1. How much of each factor will each firm use? What is the profit maximizing level of output? Is this firm making a profit or a loss? How much is the profit or loss?